97. *Note on Left Simple Semigroups*

By Tôru SAITÔ

Tokyo Gakugei University

(Comm. by K. SHODA, M.J.A., Oct. 12, 1959)

1. Teissier [5] considered homomorphisms of a left simple semigroup with no idempotent onto a semigroup which contains at least one idempotent, and characterized the inverse images of idempotents in such homomorphic mappings. In this note, we consider a method of constructing such a homomorphism, which turns out to be the finest in such homomorphisms.

We use terminologies in [2] without definitions and use the results obtained in [2] and [3] freely.

2. In this note, we denote a left simple semigroup by S.

In S, we define a binary relation ${}_S\Sigma$ as follows:

for $a, b \in S$, $a \equiv b({}_S\Sigma)$ means that there exists a finite sequence of elements $m_1, \cdots, m_{n-1}$ such that

$$aS \between m_1 S \between \cdots \between m_{n-1} S \between bS,$$

where $xS \between yS$ signifies that the sets xS and yS have at least one element in common.

It is easy to see that ${}_S\Sigma$ is an equivalence relation in S, which is left regular, that is,

$$a \equiv b \ ({}_S\Sigma) \quad \text{implies} \quad ca \equiv cb \ ({}_S\Sigma).$$

In Dubreil's terminology, ${}_S\Sigma$ is the generalized left reversible equivalence associated to S [1, p. 258].

Lemma 1. *$ac \equiv a({}_S\Sigma)$ for all $a, c \in S$* (cf. [1, p. 260, Théorème 8]).

Proof. For any $s \in S$, we have $(ac)s = a(cs)$. Hence we have

$$acS \between aS, \text{ and so } ac \equiv a({}_S\Sigma).$$

Lemma 2. *${}_S\Sigma$ is an equivalence relation which is regular.*

Proof. It suffices to show that ${}_S\Sigma$ is right regular, that is, $a \equiv b({}_S\Sigma)$ implies $ac \equiv bc({}_S\Sigma)$. And, in fact, if $a \equiv b({}_S\Sigma)$, then, by Lemma 1, we have $ac \equiv a \equiv b \equiv bc({}_S\Sigma)$.

Now, we denote the core of S by I. I is a normal and left unitary subsemigroup of S [2, Theorem 1].

Lemma 3. *For any $a \in S$, there exists an element $i \in I$ such that $a \equiv i({}_S\Sigma)$.*

Proof. Since S is left simple, we can take an element u such that $ua = a$. u is clearly an element of I, and also, by Lemma 1, we have $a = ua \equiv u({}_S\Sigma)$.

Since ${}_S\Sigma$ is a regular equivalence relation in S, ${}_S\Sigma$ induces a regular equivalence relation in semigroup I. Hence we can consider