95. *Remarks on a Theorem concerning Conformal Transformations*

By Yoshihiro TASHIRO
Department of Mathematics, Okayama University, Okayama, Japan
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In a recent paper K. Yano and T. Nagano [3] proved the following

Theorem A. *Let M be a complete Einstein manifold and suppose that a vector field on M generates globally a one-parameter group of non-homothetic conformal transformations.*[1] *Then M is isometric to a spherical space, i.e. a simply connected, complete space of positive constant sectional curvature. In particular M is homeomorphic to the sphere S^n.*

On the other hand, S. Ishihara and the present author [1] investigated the topological and differential-geometrical properties of compact or complete Riemannian manifolds admitting a concircular transformation. A concircular transformation of a Riemannian manifold M with metric $g_{\mu\lambda}$ into a Riemannian manifold $'M$ with metric $'g_{\mu\lambda}$ is by definition a conformal transformation

$$'g_{\mu\lambda}=\rho^2 g_{\mu\lambda}, \tag{1}$$

which carries geodesic circles in M to geodesic circles in $'M$, and is characterized by the equation

$$\nabla_\mu \rho_\lambda - \rho_\mu \rho_\lambda = \psi g_{\mu\lambda}, \tag{2}$$

where ρ is a positive-valued function on M, $\rho_\lambda=\nabla_\lambda \log \rho$ and ψ is a function on M. We obtained the following theorems:

Theorem B. *Let M and $'M$ be Riemannian manifolds whose scalar curvatures k and $'k$ are constant. We assume that M is complete and that there exists a concircular transformation of M into $'M$. Then the manifold M is*

I) *a Euclidean space, if $k=0$,*

II) *a spherical space, if $k>0$, or*

III) *a hyperbolic space, if $k<0$.*[2]

Theorem C. *In addition to the assumptions of Theorem* B, *assume that $'M$ is complete too and the concircular transformation*

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1) In this paper we suppose that manifolds are always connected, of dimension $n>2$ and of class C^∞, and that the differentiability of transformations and quantities is also of class C^∞. Greek indices run from 1 to n. We shall deal only with non-homothetic conformal transformations, and the term "conformal" will always mean "non-homothetic conformal".

2) The scalar curvatures in this paper are different from those in [7] in the sign.