140. *On Probabilities of Non-Paternity with Reference to Consanguinity. II*

By Yûsaku KOMATU and Han NISHIMIYA

Department of Mathematics, Tokyo Institute of Technology and
Department of Mathematics, Shibaura Institute of Technology

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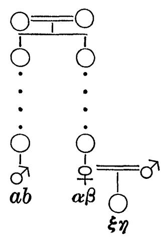
3. **Non-paternity of a putative man related with the mother but not with the father.** We now proceed to the case where a putative man and the mother of a child have antecedants of μth and νth generations respectively in common while the true father of the child is in no consanguineous relation with them. The probability of the triple which consists of a putative man A_{ab} and a mother-child combination $(\alpha\beta;\xi\eta)$ under the imposed relationship is given by



$$\bar{A}_{\alpha\beta}V(\alpha\beta;\xi\eta\mid\sigma_{\mu,\overset{♀}{\nu+1}})=\sum_{\Omega}\sigma_{\mu\nu}(ab,\alpha\beta),$$

where the range of summation is, as before, the set $\Omega=\Omega(\alpha\beta;\xi\eta)$ of types A_{ab} which together with $A_{\alpha\beta}$ can not produce $A_{\xi\eta}$. Now fortunately here also, it can be shown directly that there exists a remarkable identity

$$V(\alpha\beta;\xi\eta\mid\sigma_{\mu,\overset{♀}{\nu+1}})=(1-2^{-\lambda+1})V(\alpha\beta;\xi\eta)$$

provided $\lambda=\mu+\nu-1>1$, while for the exceptional value $\lambda=1$, i.e. $\mu=\nu=1$ we have

$$V(\alpha\beta;\xi\eta\mid\sigma_{1,\overset{♀}{2}})=\frac{1}{4}V(\alpha\beta;\xi\eta).$$

Consequently, subsequent arguments can be economized and really reduced to those in the ordinary case without any consanguinity. The final result for the total probability of non-paternity in the present case is given by

$$P(\sigma_{\mu,\overset{♀}{\nu+1}})=\begin{cases}\dfrac{1}{4}P & (\mu=\nu=1),\\ (1-2^{-\lambda+1})P & (\lambda=\mu+\nu-1>1).\end{cases}$$

The decrement of $P(\sigma_{\mu,\overset{♀}{\nu+1}})$ compared with P as well as its behavior as $\lambda\to\infty$ is quite similar as in the previous case. In particular, we now have

$$P-P(\sigma_{\mu,\overset{♀}{\nu+1}})=\begin{cases}\dfrac{3}{2}(P-P(\sigma_{\mu,\overset{♂}{\nu+1}})) & (\mu=\nu=1),\\ 2\,(P-P(\sigma_{\mu,\overset{♂}{\nu+1}})) & (\mu+\nu>2)\end{cases}$$

and

$$P(\sigma_{\mu,\overset{♀}{\nu+1}})<P(\sigma_{\mu,\overset{♂}{\nu+1}})<P(\sigma_{\mu\nu;1})<P \qquad (\mu+\nu\geqq 2).$$

4. **Illustrative examples.** The general results obtained in the