135. *Mappings and Pseudo-compact Spaces*

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Let f be a mapping of a topological space X onto another topological space Y: then, by Whyburn [1, 2], Stone [3], Morita [4–6], Hanai [6–8], McDougle [9, 10] and others, it is known that some properties of f, for instance closedness, openness and quasi-compactness, give the interest relations between X and Y.

In this paper, we shall first prove that a space X is pseudo-compact if and only if any continuous mapping of X onto a weakly separable T_2-space is always a P_0-mapping. Next we shall show, for a continuous mapping f of a pseudo-compact space X onto a weakly separable T_2-space, that 1) f is quasi-compact if and only if $f(\mathfrak{B}U)=\mathfrak{B}f(U)$ for any open inverse subset U where $\mathfrak{B}U$ denotes the boundary of U, and 2) if $\mathfrak{B}f^{-1}(y)$ is compact for every $y\in Y$ and X is locally compact, then f is always closed and Y is locally compact. Finally we give some characterization of compact spaces.

In the following, we assume that any mapping is always continuous. Let f be a mapping of X onto Y where X and Y are topological spaces; f is a P_1 (or P_0)-*mapping* provided that whenever $y\in Y$ and U is any neighborhood of $f^{-1}(y)$, $y\in \operatorname{Int} f(U)$ (or $y\in \operatorname{Int} f(\overline{U})$). f is a P_2-*mapping* if for each $y\in Y$, there is a compact subset C of $f^{-1}(y)$ such that $\operatorname{Int} f(U)\ni y$ for every open subset U containing C (the definitions of both P_1 and P_2-mappings are due to McDougle [9]). f is called to have a *compact trace property* [2] if any point y of Y is interior of the image of some compact subset of X. The following implications are obvious: (*open* $\rightarrow P_1$), (*closed* $\rightarrow P_1\rightarrow$ *quasi-compact*) and ($P_2\rightarrow P_1\rightarrow P_0$).

1. Characterizations of pseudo-compact spaces. The following lemma is useful.

Lemma 1. *Let f be a mapping of a topological space X onto a T_2-space Y. If $\{y_n\}\rightarrow y$ in Y and x_n is any point contained in $f^{-1}(y_n)$, then $\overline{\{x_n\}}-\{x_n\}\ni x$ implies $f(x)=y$.*

Theorem 1. *The following conditions are equivalent for a complete regular T_1-space X;*

1) *X is pseudo-compact.*
2) *Any mapping of X onto a weakly separable T_2-space is P_0.*
3) *If f is a mapping of X onto a weakly separable T_2-space Y, then*