4. On Predicates Expressible in the 1-Function Quantifier Forms in Kleene Hierarchy with Free Variables of Type 2*)

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The aim of this note is to show that there are certain non analogical properties between the 1-function quantified predicates (or the sets definable by such predicates) in Kleene hierarchy with free variables of types ≤ 1 and those with free variables of type 2.

In particular, there is no second separation (and hence, reduction) result for the classes of sets definable by the 1-function quantified predicates in Kleene hierarchy with free variables of type 2, in contrast to the case with free variables of types ≤ 1[1] in which the reduction theorem holds for the universal quantifier first side — namely, using the notation of Addison's [1], for Π_1^1 sets (and hence, as to separation, the first and the second separation theorems hold for the existential quantifier first side — namely, for Σ_1^1 sets).[2]

Throughout this paper, the ideas and the techniques of Kleene [8, in particular, 8.2–8.8] are used.

1. We assume familiarity with [5–8], and use the notation of them unless further references. Using the notation of [1] (in addition, with superscripts $m=0, 1, 2, \cdots$ exhibiting the maximal type of free variables of the predicates), we denote by $\Sigma_k^{r,m}$, $\Pi_k^{r,m}$ the classes of predicates (or sets) in Kleene hierarchies, according to the kind of the outermost quantifier, the highest type "r" of variables quantified, the number "k" of alternations of the quantifiers of type r and the highest type "m" of quantifier-free variables in the forms of Kleene's tables (cf. [6, (a) with free variables of types ≥ 0, p. 315] and [8, (c_1) p. 41]), respectively. For example, a predicate $P(a, \alpha, \mathbf{F})$ expressible in the form

*) The contents with detailed proofs will appear in a forthcoming paper. As to Kleene hierarchies, it is referred to Kleene's excellent series of papers [4, 6–8] on hierarchies obtained by quantifying variables of recursive predicates.

1) Cf. [1, §5, Case 2].

2) Recently Addison [2, p. 352] has conjectured that on hierarchies based on quantification of higher type, under the assumption of the axiom of constructibility, the separation results are uniform according to the kind of the outermost quantifier for all types r ($r\geq 0$) and levels k ($k\geq 1$) except for the lone case when $r=1$, $k=1$. In fact, for Kleene hierarchies based on quantification of finite type, we can assure that Addison's conjecture holds, and see that the separation results are uniform also in types m ($m\leq r+1$) of quantifier-free variables under the same exception. As to the case when $r=1$, $k=1$, the author's result shows that the separation principle does not uniformly behave to types of quantifier-free variables.