3. Certain Generators of Non-hyperelliptic Fields of Algebraic Functions of Genus $\geqq 3$

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Let Ω be an algebraically closed field of characteristic 0, and K a field of algebraic functions of one variable over Ω whose genus will be denoted by G. We shall denote the elements of K by letters like x_i, x, y, u, u', v; the divisors by E_i, prime divisors by P, the divisor classes of E_i by $\bar{E}_i$. The divisor classes of degree 0 form a group, which becomes the Jacobian variety of K when Ω is the field $\boldsymbol{C}$ of complex numbers. We shall consider the elements of this group whose orders are finite and divide 2. They will be called *two-division points* of K. They form a group $\mathfrak{g}$ isomorphic to the direct sum of $2G$ cyclic groups of order 2, so that there are 2^{2G} two-division points $\bar{E}_i$, $1\leqq i\leqq 2^{2G}$, of K (cf. [1, p. 176, Th. 16 and Cor. to Th. 16] and [2, p. 79]). Let E_i be *arbitrary* representatives of $\bar{E}_i$, $1\leqq i\leqq 2^{2G}$, and x_i an element of K such that $(x_i)=E_i^2$. Now we consider the subfield

$$k=\Omega(x_1,\cdots,x_{2^{2G}})$$

of K. We shall show in Theorem 1 that $K=k$ (i.e. that K is generated by the functions x_i determined by two-division points $\bar{E}_i$ if K is not hyperelliptic and $G\geqq 3$, and in Theorem 2 that $[K:k]=1$, 2 or 4 if K is hyperelliptic.

The above notations will be used throughout the paper. The genus of k will be denoted by g. We put $[K:k]=n$.

LEMMA. *If* $n>1$ *and* $G\geqq 2$, *then* $g=0$ *and* $n\leqq 2+\dfrac{1}{G-3/2}$.

PROOF. We use Riemann-Hurwitz's formula:

$$(1)\qquad 2G-2=n(2g-2)+\sum_P(e_P-1),$$

where P runs over the prime divisors of K and e_P is the ramification index of P. We recall first, that $G>g$ since $G\geqq 2$, and that the number of 2-division points of k is 2^{2g}. Denote by $(x_i)_K$ and $(x_i)_k$ the divisors of x_i in K and k respectively. We have

$$(x_i)_K=E_i^2=\mathrm{Con}_{k/K}(x_i)_k.$$

Now every divisor $(x_i)_k$ is either a square of another divisor: $(x_i)_k=e_i^2$ or not a square of any divisor: $(x_i)_k=e_i$; but we can show here that at most 2^{2g} divisors $(x_i)_k$ are squares of other divisors; in fact, if $(x_i)_k=e_i^2$, then e_i represents a 2-division point of k, and it follows from