1. On the Thue-Siegel-Roth Theorem. III

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1. It is well known that a classical theorem of J. Liouville on the rational approximation to real algebraic numbers has an analogue in algebraic function fields and, as was shown by K. Mahler [2], the analogue of Liouville's theorem for function fields cannot be improved if the field of constants is of positive characteristic. On the other hand, as the theorem of Liouville for algebraic numbers has been improved by A. Thue, C. L. Siegel and K. F. Roth,[1] a similar improvement is possible in the case of the analogue of Liouville's theorem for algebraic functions when the constant field is of characteristic 0. Indeed, B. P. Gill [1] has proved for function fields with the constant field of characteristic 0 an analogue of the Thue-Siegel theorem. In the present note we wish to point out that an analogue of a theorem of Roth[1] holds for function fields with the constant field of characteristic 0.

2. Let k be an arbitrary field of characteristic 0, t an indeterminate, $k[t]$ the ring of all polynomials in t with coefficients in k, and $k(t)$ the field of all rational functions in t with coefficients in k. We define a valuation $|\ |$ on $k[t]$ by putting for $a=a(t)$ in $k[t]$

$$|a|=\begin{cases} 0 & \text{if } a\equiv 0, \\ c^{\deg a} & \text{if } a\not\equiv 0, \end{cases}$$

where $c>1$ is a constant fixed throughout in the following. $k(t)$ being the quotient field of the ring $k[t]$, the valuation $|\ |$ on $k[t]$ can be extended in a natural way to $k(t)$. Thus we have defined on $k(t)$ a non-archimedean valuation which is trivial on k. The completion $k\langle t\rangle$ of $k(t)$ under this valuation is the field of all formal power series in t with coefficients in k.

Now, let K denote the set of all elements in $k\langle t\rangle$ which are algebraic over $k(t)$. Then our main result can be stated as follows:

Theorem. *Let $\alpha=\alpha(t)$ be any element of K, not identically zero. Then for each $\kappa>2$, the inequality*

$$0<\left|\alpha-\frac{p}{q}\right|<\frac{1}{|q|^{\kappa}}$$

has only finitely many solutions $p=p(t)$, $q=q(t)\not\equiv 0$ in $k[t]$.

This is a final improvement of the theorem of Gill [1].

3. We note that it is also possible to obtain an analogue of a

1) Cf. K. F. Roth: Rational approximations to algebraic numbers, Mathematika, **2**, 1-20, 168 (1955).