19. Triviality of the mod p Hopf Invariants

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In this note we shall extend Adams' result[1] to the mod p case.

1. Let p be an odd prime; let A be the Steenrod algebra over Z_p. An A-module is to be a graded left module over the graded algebra A. For each integer $k\geqq 0$, define C_k to be the free A-module generated by symbols $[\mathcal{P}^{p^i}]$ of degree $2p^i(p-1)$ $(i=0,1,\cdots,k)$ and $[\Delta]$ of degree one. C_k may be considered as a submodule of C_l for $k<l$, and the inductive limit $\bigcup_k C_k$ is denoted by C. Define $d:C\to A$ to be the A-map of degree zero such that $d[\Delta]=\Delta$ and $d[\mathcal{P}^{p^i}]=\mathcal{P}^{p^i}$ $(i=0,1,\cdots)$, where $\mathcal{P}^{p^i}$ denotes the reduced power and Δ denotes the Bockstein operator.

2. We call a homogeneous element of Ker d a *d-cycle*. A d-cycle Z may be written in such a way as $\alpha_k[\mathcal{P}^{p^k}]+\alpha_{k-1}[\mathcal{P}^{p^{k-1}}]+\cdots+\alpha_0[\mathcal{P}^1]+\alpha_\Delta[\Delta]$, of which $\alpha_k[\mathcal{P}^{p^k}]$ $(\alpha_k\neq 0)$ is called the leading term of Z.

We choose specific d-cycles (occasionally indicated only by their leading terms) as follows:

$$U_0=\Delta[\Delta],\quad V_0=(2\mathcal{P}^1\Delta-\Delta\mathcal{P}^1)[\mathcal{P}^1]-2\mathcal{P}^2[\Delta],\quad W_0=\mathcal{P}^{p-1}[\mathcal{P}^1],$$
$$Z_k=\Delta[\mathcal{P}^{p^k}]+\cdots \qquad (k\geqq 1),$$
$$Z_{i,k}=\mathcal{P}^{p^i}[\mathcal{P}^{p^k}]+\cdots \qquad (0\leqq i\leqq k-2),$$
$$U_k=\mathcal{P}^{2p^{k-1}}[\mathcal{P}^{p^k}]+\cdots \qquad (k\geqq 1),$$
$$V_k=c(2\mathcal{P}^{p^k+p^{k-1}}-\mathcal{P}^{p^k}\mathcal{P}^{p^{k-1}})[\mathcal{P}^{p^k}]+\cdots \qquad (k\geqq 1),$$
$$W_k=c(\mathcal{P}^{p^k(p-1)})[\mathcal{P}^{p^k}]+\cdots \qquad (k\geqq 1),$$

where c is the conjugation.[2] We call these *basic* d-cycles.

Lemma. *$C_k\frown$Ker d is generated by the basic d-cycles as an A-module.*

This lemma follows from Proposition 1.7 of Toda's paper.[3]

To each basic d-cycle Z corresponds a *basic* (stable secondary cohomology) operation Φ_Z. Among the basic secondary operations, only the followings are of degree even:

Φ_{V_0}, of degree $4(p-1)$, and Φ_{Z_k}, of degree $2p^k(p-1)$ $(k\geqq 1)$.

3. We shall state a proposition which is a generalization of

1) J. F. Adams: On the non existence of elements of Hopf invariant one, Bull. Amer. Math. Soc., **64**, 279-282 (1958).

2) J. Milnor: The Steenrod algebra and its dual, Ann. of Math., **67**, 150-171 (1958).

3) H. Toda: p-primary components of homotopy groups, I. Exact sequences in Steenrod algebra, Memoirs of the College of Sci., Univ. of Kyoto, ser. A, **31**, Math., no. 2, 129-142 (1958); II. mod p Hopf invariant, ibid., **31**, 143-160 (1958).