18. A Continuity Theorem in the Potential Theory

By Masanori KISHI
Mathematical Institute, Nagoya University
(Comm. by K. KUNUGI, M.J.A., Feb. 12, 1960)

Introduction. Let Ω be a locally compact separable metric space and let Φ be a positive symmetric kernel satisfying the continuity principle, that is, let Φ be a real-valued continuous function defined on the product space $\Omega\times\Omega$ such that

1° $0<\Phi(P,Q)\leq+\infty$,

2° $\Phi(P,Q)$ is finite except at most at the points of diagonal set of $\Omega\times\Omega$,

3° $\Phi(P,Q)=\Phi(Q,P)$,

4° for any compact set $K\subset\Omega$ and for any positive number ε, there is a compact set L such that

$$\Phi(P,Q)<\varepsilon \quad \text{on } K\times(\Omega-L),$$

5° if a potential U^{μ} of a positive measure μ with compact support S_{μ} is finite and continuous as a function on the support S_{μ}, then it is continuous in Ω, where the potential U^{μ} is defined by

$$U^{\mu}(P)=\int\Phi(P,Q)d\mu(Q).$$

It is known that every potential U^{μ} of a positive measure with compact support is quasi-continuous in Ω, that is, for any positive number ε, there is an open set G_{ε} such that $\operatorname{cap}(G_{\varepsilon})\leq\varepsilon$ and U^{μ} is finite and continuous as a function on $\Omega-G_{\varepsilon}$. This is called an "in the large" continuity theorem. In this note we communicate an "in the small" continuity

Theorem. *Let μ be a positive measure with compact support. Then at any point P except at most at the points of a polar set, there exists an open set $G(P)$, thin at P, such that the restriction of U^{μ} to $\Omega-G(P)$ is finite and continuous at P.*

This theorem was proved by Deny [3] in the case of the Newtonian potentials in the m-dimensional Euclidean space. Recently Smith [6] has remarked that this is valid for the potentials of order α, $0<\alpha<m$.

1. **Capacities.** A set $E\subset\Omega$ is called a *polar set* if it is contained in some $I_{\mu}=\{P: U^{\mu}(P)=+\infty\}$, where μ is a positive measure of total measure finite. We denote by $\mathfrak{P}$ the family of all polar sets. For any set X we put

$$\mathfrak{F}_X=\{\mu\geq0;\ \mu(\Omega)<+\infty,\ U^{\mu}\geq1 \text{ on } X \text{ except } E\in\mathfrak{P}\},$$

$$f(X)=\begin{cases}\inf_{\mu\in\mathfrak{F}_X}\mu(\Omega) & \\ +\infty & \text{if } \mathfrak{F}_X \text{ is empty,}\end{cases}$$