17. *Note on Finite Semigroups which Satisfy Certain Group-like Condition*

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§1. **Introduction.** In this note we shall report promptly some results about $\mathfrak{S}$-semigroups and $\mathfrak{H}$-semigroups without proof. The propositions will be precisely discussed in another papers [3, 4].

A finite semigroup S is said to have $\mathfrak{S}$-property if S of order n contains no proper subsemigroup of order greater than $n/2$. We mean by a decomposition of S a classification of the elements into some classes due to a congruence relation. A decomposition is called homogeneous if each class is composed of equal number of elements. If every decomposition of a finite semigroup S is homogeneous, we say S has $\mathfrak{H}$-property, or S is called a $\mathfrak{H}$-semigroup.

According to Rees [1], if a finite semigroup S is simple, it is represented as a regular matrix semigroup with a ground group G and with a defining matrix $P=(p_{ji})$ of type (l, m), namely

either $\quad S=\{(x;\ i\ j)\mid x\in G,\ i=1,\cdots,m;\ j=1,\cdots,l\}$

or $\quad S=\{(x;\ i\ j)\mid x\in G,\ i=1,\cdots,m;\ j=1,\cdots,l\}\smile\{\mathbf{0}\}$

in which $\mathbf{0}$ is the two-sided zero of S. The multiplication is defined as

$$(x;\ i\ j)(y;\ s\ t)=\begin{cases}(xp_{js}y;\ i\ t) & \text{if } p_{js}\neq 0\\ \mathbf{0} & \text{if } p_{js}=0 \text{ and hence } S \text{ has } \mathbf{0}.\end{cases}$$

Let $M=\{1,\cdots,m\}$, $L=\{1,\cdots,l\}$. M and L are regarded as a right-singular semigroup and a left-singular semigroup respectively. For the sake of convenience, the notations

$$\text{Simp.}(G; P) \quad \text{and} \quad \text{Simp.}(G, 0; P)$$

denote simple semigroups S with a ground group G and with a defining matrix P. The former is one without zero, whence $p_{ji}\neq 0$ for all i, j, but the latter denotes one with zero $\mathbf{0}$, so that if $p_{ji}\neq 0$ for all i and j, S contains no zero-divisor.

§2. **$\mathfrak{S}$-semigroups.** The following $\mathfrak{S}_1$-property is stronger than $\mathfrak{S}$-property, i.e. $\mathfrak{S}_1$-property implies $\mathfrak{S}$-property.

A finite semigroup S is said to have $\mathfrak{S}_1$-property if the order of any subsemigroup is a divisor of the order of S.

Let e be a unit of a finite group G.

Lemma 2.1. $\text{Simp.}\left(G; \begin{pmatrix} e \\ e\end{pmatrix}\right)$ *is an $\mathfrak{S}_1$-semigroup.*

Lemma 2.1′. $\text{Simp.}(G; (e\ e))$ *is an $\mathfrak{S}_1$-semigroup.*