36. *A Remark on my Paper "A Unique Continuation Theorem of a Parabolic Differential Equation"*

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(Comm. by K. KUNUGI, M.J.A., March 12, 1960)

§1. Introduction. It is well known that real solutions of second order elliptic equations with real coefficients have the property that if the difference of two vanishes sufficiently fast at a point then they are identical in their common range of definition. The question naturally arises what kind of extensions of the unique continuation theorem mentioned above are valid for solutions of parabolic differential equations?

In the present note, we give a simple proof of the theorem[1] in my paper[2] in which I asserted a partial answer of the problem.

§2. Let G be a convex domain of the Euclidean $n+1$ space $R_{t,x}:\{-\infty<t<+\infty,\ -\infty<x_i<+\infty\ (i=1,2,\cdots,n)\}$, containing a curve $C:\{(t,x_i(t))\,|\,t\in[a,b]\}$, where $x_i(t)\in C^1[a,b]$.

Consider real solutions u of an inequality of the following kind:

$$(2.1)\quad \left|\frac{\partial u(t,x)}{\partial t}-\sum_{i,j=1}^{n}a_{ij}(t,x)\frac{\partial^2 u(t,x)}{\partial x_i\partial x_j}\right|\leqq M\left\{\sum_{i=1}^{n}\left|\frac{\partial u(t,x)}{\partial x_i}\right|+|u(t,x)|\right\}.$$

Here $(a_{ij}(t,x))$ denote a positive definite, symmetric matrix of real valued functions $a_{ij}(t,x)\in C^2(G)$,[3] and M a constant.

The theorem in my previous paper is the following.

Theorem.[4] *If u is a solution of* (2.1) *in the domain G and if for any $\alpha>0$*

$$(2.2)\quad \lim_{r\to 0}\max_{\substack{|x-x(t)|=r\\ t\in[a,b]\\ i,j=1,2,\cdots,n}}\left\{\left|u(t,x)\right|,\left|\frac{\partial u}{\partial x_i}(t,x)\right|,\left|\frac{\partial^2 u}{\partial x_i\partial x_j}(t,x)\right|\right\}|x-x(t)|^{-\alpha}=0,$$

then u vanishes identically in the horizontal component $G\wedge\{(t,x)\,|\,t\in[a,b]\}$.

In the following we shall sketch the direct proof of the theorem using notations stated in my paper without repeating definitions of them.

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1) See below §2.

2) T. Shirota: A unique continuation theorem of a parabolic differential equation, Proc. Japan Acad., **35**, 455-460 (1959).

3) This restriction of the coefficients may be weakened. For instance, we may remove the restriction with respect to $a_{ij}|_{tt}$.

4) More precisely, in my previous paper we assume that $x_i(t)\in C^2[a,b]$ and in (2.2) the term with respect to u_t was inserted, but the refinements of these assumptions in the theorem will be of no essential matter.