27. On the mod p Hopf Invariant

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J. F. Adams [1] has proved that there is no element of Hopf invariant one in $\pi_{2n-1}(S^n)$ $(n\geqq 16)$.

In other words, his result may be expressed as follows:

If $p=2$, mod p Hopf invariant homomorphism

$$H_p:\pi_{m+n-1}(S^m)\to Z_p,\quad n=2t(p-1)$$

is trivial for $t\geqq p^3$.

In case of mod p (p: odd prime), we have the following

Theorem 1. *If p is an odd prime, the* mod p *Hopf invariant homomorphism is trivial for $t\geqq p$.*

The special case of this theorem, corresponding to $t=p$ was proved by Toda [2].

We shall adopt the definition of the stable secondary cohomology operation of Adams [1]. Then we have a similar result to the theorem of Adams [1] on Sq^{2^k} $(k\geqq 4)$.

Theorem 2. *$\mathcal{P}^{p^k}$ $(k\geqq 1)$ can be represented in the form $\sum a_i\Phi_i$ where Φ_i are stable secondary cohomology operations and a_i are elements of Steenrod algebra with positive degrees.*

Theorem 1 is easily deduced from Theorem 2. The special case of Theorem 2 for $k=1$ was also proved by Toda [2, 3].

We shall denote the Steenrod algebra over Z_p by A and denote the A free module with the symbolic base $[c(\Delta)]$, $[c(\mathcal{P}^1)],\cdots,[c(\mathcal{P}^{p^k})]$ by C_1^k $(k\geqq 0)$. Moreover, define the element $z_{-1,k}$ $(k\geqq 1)$ of C_1^k as follows:

$$z_{-1,k}=c(\Delta)[c(\mathcal{P}^{p^k})]-c(\Delta,\mathcal{P}^{p^k-1})[c(\mathcal{P}^1)]-c(\mathcal{P}^{p^k})[c(\Delta)],$$

where Δ is the Bockstein operator associated with the exact sequence $0\to Z_p\to Z_{p^2}\to Z_p\to 0$ and c is the conjugacy operation [2]. Let d be the A-homomorphism of C_1^k into $A=C_0$ such that $d[c(\Delta)]=c(\Delta)$, $d[c(\mathcal{P}^{p^i})]=c(\mathcal{P}^{p^i})$, $i=0,1,\cdots,k$. Then $z_{-1,k}$ is a d-cycle, i.e. $d(z_{-1,k})=0$. The stable secondary cohomology operation associated with $(d, z_{-1,k})$ will be denoted with $\Phi_{z_{-1,k}}$. This is uniquely determined [1, Theorem 3]. Let ε be the augmentation (A-homomorphism) of A into $H^+(X,Z_p)=\sum_{i>0}H^i(X,Z_p)$ which maps A free base 1 into an element u of $H^q(X,Z_p)$. Then we have $\varepsilon d=0$, if $u\in\bigcap_{i=0}^k\mathrm{Ker}\,c(\mathcal{P}^{p^i})\frown\mathrm{Ker}\,c(\Delta)=\bigcap_{i=0}^k\mathrm{Ker}\,\mathcal{P}^{p^i}\frown\mathrm{Ker}\,\Delta$, in which case $\Phi_{z_{-1,k}}(u)$ is defined.

Consider the effect of $\Phi_{z_{-1,k}}$ for element $y^{p^{k+1}n}$ in $H^{2p^{k+1}n}(P,Z_p)$, where P is infinite dimensional complex projective space and y is a