23. *Cosheaves*

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In this note we shall define a cosheaf on a paracompact space X, which is a dual concept of a sheaf (§1). If the base space X is a compact Hausdorff space we can develop a homology theory of X with coefficients in a cosheaf (§2). This homology theory is equivalent to the Čech homology theory and is dual to the cohomology theory with coefficients in a sheaf (§3).[1]

1. Let X be a paracompact space. We denote by $\mathfrak{A}(X)$ the family of all closed subsets of X. Let us suppose that a compact topological additive group $\mathfrak{F}(A)$ is associated with each $A\in\mathfrak{A}(X)$. In particular, let $\mathfrak{F}(\phi)=\{0\}$. For each pair (A, B) $(A, B\in\mathfrak{A}(X)$ and $A\supset B)$ let a continuous homomorphism $\iota_{A,B}$ of $\mathfrak{F}(B)$ into $\mathfrak{F}(A)$ be defined such that (i) $\iota_{A,A}$ is the identity mapping for each $A\in\mathfrak{A}(X)$, and (ii) $\iota_{A,C}=\iota_{A,B}\circ\iota_{B,C}$ holds for $A, B, C\in\mathfrak{A}(X)$ and $A\supset B\supset C$. Moreover, let $\mathfrak{U}(A)$ $(A\in\mathfrak{A}(X))$ be the family of all $B\in\mathfrak{A}(X)$ such that A is contained in the interior of B. Then $\mathfrak{U}(A)$ is a directed family of sets with respect to the inclusion relation and $\{\mathfrak{F}(B); B\in\mathfrak{U}(A)\}$ is an inverse system of compact additive groups with respect to the continuous homomorphisms $\{\iota\}$. Let us suppose further that

$$(1) \qquad \mathfrak{F}(A)=\text{inv lim}\,\{\mathfrak{F}(B); B\in\mathfrak{U}(A)\} \qquad \text{for } A\in\mathfrak{A}(X)$$

hold. Then we call the system $\mathfrak{F}=\{\mathfrak{F}(A), \iota_{A,B}\}$ a *precosheaf* with the base space X.[2] If necessary we denote $\iota^{\mathfrak{F}}_{A,B}$ instead of $\iota_{A,B}$. In the following we fix a base space X.

A precosheaf $\mathfrak{G}=\{\mathfrak{G}(A), \iota^{\mathfrak{G}}_{A,B}\}$ is called a *subprecosheaf* if (i) for each $A\in\mathfrak{A}(X)$ $\mathfrak{G}(A)$ is a closed subgroup of $\mathfrak{F}(A)$ with the relative topology, (ii) $\iota^{\mathfrak{G}}_{A,B}=\iota^{\mathfrak{F}}_{A,B}\,|\,\mathfrak{G}(B)$ for $A\supset B$ holds and (iii) $\mathfrak{G}(A)=\text{inv lim}\,\{\mathfrak{G}(B); B\in\mathfrak{U}(A)\}$ holds for each $A\in\mathfrak{A}(X)$. Let $\mathfrak{G}$ be a subprecosheaf of a precosheaf $\mathfrak{F}$. Let us put $\mathfrak{H}(A)=\mathfrak{F}(A)/\mathfrak{G}(A)$ with the quotient topology for each $A\in\mathfrak{A}(X)$ and let the homomorphism $\iota^{\mathfrak{H}}_{A,B}$ be induced from $\iota^{\mathfrak{F}}_{A,B}$. Then $\mathfrak{H}=\{\mathfrak{H}(A), \iota^{\mathfrak{H}}_{A,B}\}$ is a precosheaf. We call $\mathfrak{H}$ the *quotient precosheaf* of $\mathfrak{F}$ by $\mathfrak{G}$.

Let $\mathfrak{F}, \mathfrak{G}$ be two precosheaves. Let φ_A be a continuous homomorphism of $\mathfrak{F}(A)$ into $\mathfrak{G}(A)$ for each $A\in\mathfrak{A}(X)$ and let us assume that

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1) In this note we shall only sketch our results. The details and further developments will be discussed in another paper.

2) This definition is dual to that of a sheaf used in Cartan [1], XII: Faisceaux et carapaces.