65. Adjoint Space and Dual Space

By Toshio SHIBATA
Tokyo University of Education

(Comm. by K. KUNUGI, M.J.A., May 19, 1960)

1. Introduction. Concerning the relation between a locally convex space E and its dual space E' (the totality of continuous linear functionals), the conditions of the semi-reflexivity, the reflexivity, etc. are well known, under the foundation of Mackey's theorem (for ex. [1]). On the other hand, as a space of linear functionals on E, there is a method to consider its *adjoint space* $\overline{E}$ (the totality of linear functionals which are bounded on each bounded set in E) and the condition of reflexivity is known [2].

In this paper, we consider the relation between the adjoint space $\overline{E}$ and the dual space E' of a locally convex separative space E, especially, we give a theorem of Mackey's type with respect to the adjoint space (THEOREM 1).

For a locally convex separative topology T of a linear space E, its adjoint space and its dual space are denoted by $\overline{(E;T)}$ and $(E;T)'$, respectively. Two locally convex topologies T_1 and T_2 of E are said to be *equivalent with respect to bounded set*, when the concepts of the boundedness under T_1 and T_2 are identical, and this equivalence is denoted by $T_1 \overset{b}{\sim} T_2$.

2. A theorem of Mackey's type. Let (E, F) be a separative dual system of two linear spaces. The necessary and sufficient condition that a locally convex separative topology T on E is compatible with the dual system (E, F), that is, $(E;T)'=F$, is that T is stronger than the topology $\sigma(E, F)$ and weaker than the Mackey's topology $\tau(E, F)$ (theorem of Mackey). The following theorem is one of this type concerning the adjoint space.

THEOREM 1. *Let (E, F) be a separative dual system of two linear spaces. The necessary and sufficient condition that F is the adjoint space of E with a locally convex separative topology T on E, or $\overline{(E;T)}=F$ symbolically, is that:*

1. $T \overset{b}{\sim} \sigma(E, F)$
2. *the Mackey's topology $\tau(E, F)$ is bornologic.*

PROOF. Necessity. We denote by $\mathfrak{A}$, the totality of bounded sets under the topology T, and consider a topology T_0 on E defined by all disks which absorb each bounded set under the topology T. T_0 is