63. *On the Semi-exact Canonical Differentials of the First Kind*

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1. **Introduction.** Let R be an arbitrary open Riemann surface of finite genus g. We shall denote by $\mathfrak{R}$ the class of semi-exact canonical differentials[1] (or integrals of these) on R, and by δ an arbitrary divisor of finite order $d[\delta]$ on R. Then, with differentials and integrals (functions) of $\mathfrak{R}$, the following Riemann-Roch's theorem was established by Prof. Kusunoki:[2]

$$A[\delta^{-1}]-B[\delta]=2(d[\delta]-g+1),$$

where $A[\delta^{-1}]$ denotes the number of linearly independent (in the real sense) functions $\in\mathfrak{R}$, which are single-valued on R and multiples of δ^{-1}, and $B[\delta]$ the number of linearly independent differentials $\varphi\in\mathfrak{R}$ which are multiples of δ. If we take a divisor $\delta=P^r$ $(0\leqq r\leqq g,\ P\in R)$ we have therefore $B[P^r]\geqq 2(g-r)$, in particular, $B[P^g]\geqq 0$. In the present paper, we shall show that the set of the points where $B[P^g]=0$ is dense in R, and the properties of the points. Theorem 3 shows the existence of parallel slit mappings under an additional condition on the boundaries. And finally, some remarks on curves in R and points lying on the boundaries will be given.

2. **Theorem 1.** *The set of the points at which $B[P^g]=0$ is dense in R.*

Proof. If the theorem is not true, there is an open set U in R which does not contain any point where $B[P^g]=0$. Let P_0 be an arbitrary point in U. In terms of a local parameter $z=\Phi(P)(\Phi(P_0)=0)$ about P_0, each of the $2g$ basis differentials[3] $\varphi_j\in\mathfrak{R}$ $(j=1,2,\cdots,2g)$ of the first kind on R, can be represented as $\varphi_j=f_j(z)dz$, where the $f_j(z)$ $(j=1,2,\cdots,2g)$ are linearly independent analytic functions of $z=x+iy$ about P_0. We consider the following real function which is analytic with respect to x and y:

$$V_{2g}(z)\equiv|R_{2g}^0 I_{2g}^0 R_{2g}^1 I_{2g}^1\cdots R_{2g}^{g-1} I_{2g}^{g-1}|$$

where

1) Cf. Kusunoki [3, pp. 241-242], and Nevanlinna [4].
2) Kusunoki [3, Theorem 8], and Kusunoki [2].
3) The existence of these basis differentials was verified in Kusunoki [3, Theorem 1].