62. *Some Properties of Complex Analytic Vector Bundles over Compact Complex Homogeneous Spaces*

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1. This note is a summary of the author's paper which will appear in the Ôsaka Mathematical Journal on the same title. Our concerns are complex analytic vector bundles over C-manifolds in the sense of H. C. Wang [11], and, in particular, homogeneous vector bundles introduced by R. Bott [4]. Mainly by use of Bott's method, we shall investigate some properties of these bundles.

2. Let X be a C-manifold with an almost effective Klein form G/U, where G is a connected complex semi-simple Lie group and U a connected closed complex Lie subgroup of G. Now, let $E=E(\rho, F)$ denote the homogeneous vector bundle defined by a complex analytic representation (ρ, F) of U. Then, the complex vector space $\Gamma_X(E)$ of all sections of E is identified with the set of all holomorphic mappings s of G into F such that

$$s(gu)=\rho(u^{-1})\cdot s(g), \text{ for every } g\in G \text{ and } u\in U.$$

Moreover the induced representation in the sense of Bott, which we denote by $\rho^{\#}$, is defined by

$$(\rho^{\#}(g)s)(g')=s(g^{-1}g')$$

(for every $s\in\Gamma_X(E)$ and $g, g'\in G$) as a complex analytic representation of G over $\Gamma_X(E)$. We define a linear mapping ν of $\Gamma_X(E)$ into F by setting

$$\nu(s)=s(e), \quad (e=\text{the unit element of } G).$$

We say, if ν is surjective, that E has sufficiently many sections. In this case we have an exact sequence as U-modules:

$$(1) \qquad 0\longrightarrow F'\longrightarrow \Gamma_X(E)\overset{\nu}{\longrightarrow} F\longrightarrow 0$$

via $\rho^{\#}$ and ρ, as is easily verified, where F' is the kernel of ν. Now assume that $\dim F=m$ and $\dim \Gamma_X(E)=n$, and take the basis $\{\xi_1,\cdots,\xi_n\}$ of $\Gamma_X(E)$ such that $\{\xi_1,\cdots,\xi_{n-m}\}$ span F'. Then, identifying the exact sequence (1) with

$$0\longrightarrow C^{n-m}\longrightarrow C^n\longrightarrow C^m\longrightarrow 0; \quad C^m=C^n/C^{n-m},$$

we can consider $\rho^{\#}$ as a homomorphism of G into $GL(n, C)$ sending U into the subgroup $GL(n, m; C)$ which consists of non-singular matrices leaving C^{n-m} invariant. Thus, we obtain from $\rho^{\#}$, transferring to the coset spaces, a holomorphic mapping f_ρ of X into the complex Grassmann manifold $G(n, m)=GL(n, C)/GL(n, m; C)$. The last manifold