76. *On the Fundamental Theorem of the Galois Theory for Finite Factors*

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In the previous note [6], it is proved that a Galois theory holds true for finite factors under rather strong conditions. The present note will simplify the proof and clarify the relations of the previous conditions.

1. Following after the terminology of J. Dixmier [1], it is assumed that

(1) *A is a continuous finite factor acting standardly on a separable Hilbert space H.*

According to the representation theory of operator algebras (cf. [1]), (1) implies that H can be seen as the completion of the prehilbert space A^θ equipped with the usual inner product $<a^\theta \mid b^\theta>=\tau(ab^*)$ by the standard trace τ, in which 1^θ becomes the trace element of H. Under these circumstances, if G satisfies

(2) *G is an enumerable group of outer automorphisms of A,*

then the following lemma is proved as a sharpening of [6, (1)]:

LEMMA 1. *There exists a unitary representation u_g of G on H such as*

(3) $$x^g=u_g^* x u_g,$$

where x^g means the action of g on $x \in A$.

Although the lemma is proved already by I. E. Segal [7, Theorem 5.3], a sketch of the proof will be given for the sake of convenience. Naturally, the representation u_g is defined by

(4) $$a^\theta u_g = a^{g\theta}.$$

Since

$$<a^{g\theta} \mid b^{g\theta}>=\tau(a^g b^{g*})=\tau(ab^*)=<a^\theta \mid b^\theta>,$$

u_g is a unitary operator for each g. Furthermore

$$a^\theta u_g u_h = a^{g\theta} u_h = a^{gh\theta} = a^\theta u_{gh}$$

for all $a \in A$ implies that $g \to u_g$ is a unitary representation of G on H. Finally

$$b^\theta u_g^* a u_g = b^{g^{-1}\theta} a u_g = (ba^g)^\theta = b^\theta a^g$$

implies (3).

LEMMA 2. *Defining by*

(5) $$x'^g = u_g^* x' u_g \quad \text{for} \quad x' \in A,$$

G can be also considered as a group of outer automorphisms on A'.

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