98. Certain Congruences and the Structure of Some Special Bands

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1. A *band* is synonymous with an idempotent semigroup. Let S be a band, and $S\sim\Sigma\{S_\gamma:\gamma\in\Gamma\}$ its *structure decomposition* (cf. Kimura [1]). For each subset Δ of Γ, we first define the relation $\mathfrak{R}_\Delta$ on S as follows:

$$a\,\mathfrak{R}_\Delta b \text{ if and only if } \begin{cases} ab=a \text{ and both } a \text{ and } b \text{ are contained in} \\ \quad \text{the same } S_\gamma, \gamma\in\Delta, \\ \text{or} \\ ab=b \text{ and both } a \text{ and } b \text{ are contained in} \\ \quad \text{the same } S_\gamma, \gamma\notin\Delta. \end{cases}$$

Then, it is easily seen that $\mathfrak{R}_\Delta$ is an equivalence relation on S but not necessarily a congruence.

The following two theorems have been proved by Kimura [2]:

Theorem I. *$\mathfrak{R}_\phi(\mathfrak{R}_\Gamma)$, where ϕ is the empty subset of Γ, is a congruence on S if and only if S is left (right) semiregular. Further, in this case the quotient semigroup $S/\mathfrak{R}_\phi(S/\mathfrak{R}_\Gamma)$ is left (right) regular.*

Theorem II. *Both $\mathfrak{R}_\phi$ and $\mathfrak{R}_\Gamma$ are congruences on S if and only if S is regular. Further, in this case S is isomorphic to the spined product of $S/\mathfrak{R}_\phi$ and $S/\mathfrak{R}_\Gamma$ with respect to Γ.*

In this note, we shall present a necessary and sufficient condition for $\mathfrak{R}_\Delta$ to be a congruence on S, and make some generalizations of Theorems I and II. However here only the main results and necessary definitions are given, and the proofs are all omitted. We will study them in detail elsewhere.[1]

Notations and terminologies. If M and N are two sets such that $M\supset N$, then $M\setminus N$ will denote the complement of N in M. The notation ϕ will denote always the empty set. Throughout the whole paper S will denote a band, unless otherwise mentioned. The structure semilattice of S and the γ-kernel,[2] for each γ of the structure semilattice, will be denoted by Γ and S_γ respectively. And the structure decomposition of S will be denoted naturally by $S\sim\Sigma\{S_\gamma:\gamma\in\Gamma\}$. Any other notation or terminology without definition should be referred to [1].

2. Let Δ be a subset of the structure semilattice Γ of S, and

1) This is an abstract of the paper which will appear elsewhere.
2) For definition, see [1].