97. Finite-to-one Closed Mappings and Dimension. III

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The method of proof employed in the previous note [5, Theorem 4] can be applied to new characterizations[1] of dimension of metric spaces by means of a sequence of coverings, which generalize the results due to J. Nagata [7, Theorem 3] and C. H. Dowker and W. Hurewicz [2], as follows.

Theorem 1. *In order that a topological space* R *be a metrizable space with* $\dim R$[2] $\leq n$ *it is necessary and sufficient that there exists a sequence of locally finite coverings* $\mathfrak{H}_i=\{H_\alpha;\alpha\in A_i\}$, $i=1,2,\cdots$, *of* R *which satisfies the following conditions.*

(1) $\overline{\mathfrak{H}}_{i+1}=\{\overline{H}_\alpha;\alpha\in A_{i+1}\}$ *refines* $\mathfrak{H}_i$ *for every* i.

(2) $\liminf_i \operatorname{order}(x,\mathfrak{H}_i)$[3] $\leq n+1$ *for every* $x\in R$.

(3) *For any point* $x\in R$ *and any neighborhood* U *of* x *there exists* i *with* $\operatorname{Star}(x,\mathfrak{H}_i)$[4] $\subset U$.

C. H. Dowker and W. Hurewicz's characterization [2] is a direct consequence of this theorem. As a corollary of this theorem we get the following.

Theorem 2. *In order that a topological space* R *be a metrizable space with* $\dim R\leq n$ *it is necessary and sufficient that there exists a sequence of open coverings* $\mathfrak{U}_i$, $i=1,2,\cdots$, *of* R *which satisfies the following conditions.*

(1) $\mathfrak{U}^*_{i+1}$[5] *refines* $\mathfrak{U}_i$ *for every* i.

(2) $\liminf_i \operatorname{order}(x,\mathfrak{U}_i)\leq n+1$ *for every* $x\in R$.

(3) *For any point* $x\in R$ *and any neighborhood* U *of* x *there exists* i *with* $\operatorname{Star}(x,\mathfrak{U}_i)\subset U$.

J. Nagata's characterization [7, Theorem 3] is a direct consequence of this theorem.

We call a covering U of a space a multiplicative[6] one if for every non-empty intersection $\bigcap_{i=1}^{k} U_i$ of elements U_i, $i=1,\cdots,k$, of $\mathfrak{U}$ is also an element of $\mathfrak{U}$. The maximal number n such that there

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1) The detail of the content of the present note will be published in another place.

2) $\dim R$ denotes the covering dimension of R.

3) $\operatorname{order}(x,\mathfrak{H}_i)$ denotes the number of elements of $\mathfrak{H}_i$ which contain x.

4) $\operatorname{Star}(x,\mathfrak{H}_i)=\cup\{H_\alpha; x\in H_\alpha\in\mathfrak{H}_i\}$.

5) $\mathfrak{U}^*_{i+1}=\{\operatorname{Star}(U,\mathfrak{U}_{i+1}); U\in\mathfrak{U}_{i+1}\}$, where $\operatorname{Star}(U,\mathfrak{U}_{i+1})=\cup\{V; U\cap V\neq\phi$ (=the empty set), $V\in\mathfrak{U}_{i+1}\}$.

6) Cf. [1] or [7].