95. *Homological Dimension and Product Spaces*

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Let X be a topological space and A a closed subset. Let us denote by $H_n(X, A: G)$ the n-dimensional unrestricted Čech homology group of (X, A) with coefficients in an abelian group G. The *homological dimension of X with respect to G* (notation: $D_*(X: G)$) is the largest integer n such that there exists a pair (A, B) of closed subsets of X whose n-dimensional Čech homology group $H_n(A, B: G)$ is not zero. It is obvious that the relation $D_*(X: G) \leqq \dim X$ holds for any space X and any group G, where dim means the covering dimension. A topological space X is called *full-dimensional with respect to an abelian group G* in case $D_*(X: G)=\dim X$. Then the following problem arises naturally:

(*) *Given an abelian group G, what is a space which is full-dimensional with respect to the group?*

The object of this paper is to give an answer to this problem (*) in case X is a locally compact fully normal space and G belongs to a class which includes several important groups. The following theorems hold.

Theorem 1. *Let R be the additive group of all rationals. Then there exists a Cantor manifold M_0 with the property that a locally compact fully normal space X is full-dimensional with respect to R if and only if* $\dim(X \times M_0)=\dim X+\dim M_0$.

Theorem 2. *Let Q_p be the additive group of all rationals reduced* mod 1 *whose denominators are powers of a prime p. Then there exists a Cantor manifold M_p with the property that a locally compact fully normal space X is full-dimensional with respect to Q_p if and only if* $\dim(X \times M_p)=\dim X+\dim M_p$.

A sequence $\mathfrak{a}=\{q_1, q_2, \cdots\}$ is called a *k-sequence* if q_i is a divisor q_{i+1} for each i and $q_i>1$ for some i. Let us denote by Z_q the cyclic group with order q_i. There exists a natural homomorphism h_i^{i+1} from $Z_{q_{i+1}}$ onto Z_{q_i}, $i=1, 2, \cdots$. By $Z(\mathfrak{a})$ we mean the limit group of the inverse system $\{Z_{q_i}: h_i^{i+1} | i=1, 2, \cdots\}$. In a previous paper [2, p. 390], we constructed the Cantor manifold $Q(\mathfrak{a})$ for each k-sequence $\mathfrak{a}$. The following theorem is a consequence of [3, Theorem 1].

Theorem 3. *Let $\mathfrak{a}$ be a sequence. Then a locally compact fully normal space X is full-dimensional with respect to $Z(\mathfrak{a})$ if and only if* $\dim(X \times Q(\mathfrak{a}))=\dim X+\dim Q(\mathfrak{a})$.

Since the cyclic group Z_q with order q is the group $Z(\mathfrak{a})$ for the