92. *Comparability between Ramified Sets*

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Introduction. Let X be a partially ordered set and a be its element. Ub(a) and Lb(a) denote the sets $\{x \mid x \varepsilon X, x > a\}$ and $\{x \mid x \varepsilon X, x < a\}$ respectively.

DEFINITION 1. i) *A partially ordered set X is called a ramified set if* Lb(a) *is a well-ordered subset of X for any $a \varepsilon X$.* ii) *A non-void ramified set X is called perfectly irresoluble if* Ub(a) *is not totally ordered for any $a \varepsilon X$. X is called irresoluble if it includes a non-void perfectly irresoluble subset. X is called resoluble if any its non-void subset is not perfectly irresoluble.**)

In connection with Souslin's Problem (see [2]), investigations of ramified sets have been proceeded by many authors including Prof. George Kurepa (see [3]). In this paper we are interested in the internal structure of ramified sets and comparison between them, and obtained several results mentioned later, which seem fundamental in the theory of structures of them. But here we only give the outline of their proofs and details will be published elsewhere.

First the following follows from Definition 1.

THEOREM 1. *A ramified set X includes the largest* (*in the sense of inclusion*) *perfectly irresoluble subset $K(X)$. X is resoluble if and only if $K(X)$ is void.*

1. Main Theorems. Hereafter X, Y and Z denote ramified sets.

DEFINITION 2. i) *We write $X \propto Y$ if there exists a mapping f* (*many-to-one in general*) *of X into Y such that $a < x$ implies $f(a) < f(x)$, $X \sim Y$ if $X \propto Y$ and $Y \propto X$, and $X \precnsim Y$ if $X \propto Y$ and $Y \not\propto X$ where $Y \not\propto X$ is the negation of $Y \propto X$.* ii) *Let β be a regular ordinal number greater than* 0. *$\mathfrak{R}_\beta$ denotes the family of all ramified sets X with $\overline{\overline{X}} < \aleph_\beta$, and $\mathfrak{S}_\beta$ denotes the family of all resoluble sets in $\mathfrak{R}_\beta$.*

$\propto$ is a quasi-ordering and $\sim$ is an equivalence relation between ramified sets. If we identify equivalent sets, $\propto$ becomes an order relation. We shall say that X and Y are *comparable* with each other, if either $X \propto Y$ or $Y \propto X$ holds. Our Main Theorems are the following.

MAIN THEOREM A. i) *$\mathfrak{S}_\beta$ is well-ordered by $\propto$ under identifica-*

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*) Of course a void set is regarded as a well-ordered set (and hence a totally ordered set). For convenience, a void set is regarded as a ramified set which is resoluble and (perfectly) irresoluble in the same time.