116. The Mathematical Theory of the Electromagnetic Fields in Anisotropic Inhomogeneous Media

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Up to this time, theories of electromagnetic field have been studied mainly in isotropic and homogeneous media. Recently the studies of the fields in anisotropic inhomogeneous media have increased their importance in connection with the advances in microwave techniques or in other branches of electric engineerings and physics. But since the rigorous analysis of the fields in such media is so difficult and complicated, it seems to the author that there are few papers on these themes which are rigorous and general enough. In this paper, he will develop the mathematical theory of electromagnetic fields in anisotropic inhomogeneous media in its general form.

1. Any electromagnetic phenomena in a macroscopic scale are represented completely by vector functions $\boldsymbol{E}, \boldsymbol{H}, \boldsymbol{D}, \boldsymbol{B}, \boldsymbol{K}$ and a scalar function ρ, which satisfy Maxwell's equation. In addition to the equation, there are three relations between them. In the old theory in isotropic and homogeneous media, they were $\boldsymbol{D}=\varepsilon\boldsymbol{E}$, $\boldsymbol{B}=\mu\boldsymbol{H}$ and $\boldsymbol{K}=\sigma\boldsymbol{E}$ where ε, μ and σ are scalar constants characteristic of the medium. In this paper we take them as follows: $\boldsymbol{D}=[\varepsilon]\boldsymbol{E}+[\xi]\boldsymbol{H}$, $\boldsymbol{B}=[\mu]\boldsymbol{H}+[\zeta]\boldsymbol{E}$, $\boldsymbol{K}=[\sigma]\boldsymbol{E}$, where a roman letter in a bracket, such as $[\varepsilon]$, represents a 3×3 matrix, the elements $\varepsilon_{i,j}(i, j=1, 2, 3)$ of which are functions of position. The fact that the characters of the medium are represented by matrices shows that the medium is anisotropic, and that the elements of these matrices are the functions of position shows that the medium is inhomogeneous. Of course these representations include all of the isotropicity and anisotropicity, homogeneity and inhomogeneity of the medium. Because of $[\varepsilon]$ etc., it is not possible to eliminate $\boldsymbol{E}$ or $\boldsymbol{H}$ from Maxwell's equation, hence the old method of analysis is not available. A new method will be investigated in the followings.

Let ε, μ and σ be arbitrary scalar constants, and put $\varepsilon'_{ij}=\varepsilon_{ij}-\varepsilon\delta_{ij}$, $\mu'_{ij}=\mu_{ij}-\mu\delta_{ij}$, $\sigma'_{ij}=\sigma_{ij}-\sigma\delta_{ij}$. Suppose $[\varepsilon']$ is a matrix the elements of which are ε'_{ij}, then $[\varepsilon]=\varepsilon U+[\varepsilon']$, where U is the unit matrix. Similar holds for $[\mu]$ and $[\sigma]$. Hence Maxwell's equation will be reduced to

(1) $$\nabla\times\boldsymbol{E}=-i\omega\mu\boldsymbol{H}-\boldsymbol{K}_H,\quad \nabla\times\boldsymbol{H}=(\sigma+i\omega\varepsilon)\boldsymbol{E}+\boldsymbol{K}_E$$

where $\boldsymbol{K}_H=i\omega([\mu']\boldsymbol{H}+[\zeta]\boldsymbol{E})$ and $\boldsymbol{K}_E=[\sigma'+i\omega\varepsilon']\boldsymbol{E}+i\omega[\xi]\boldsymbol{H}$, when the