113. *The Lebesgue Constants for (γ, r) Summation of Fourier Series*

By Kazuo ISHIGURO
Department of Mathematics, Hokkaido University, Sapporo, Japan
(Comm. by K. KUNUGI, M.J.A., Oct. 12, 1960)

1. The Euler method of summation associates with a given sequence $\{s_n\}$ the means

$$\sigma_{n,r}=\sigma_n=\sum_{\nu=0}^{n}\binom{n}{\nu}r^{\nu}(1-r)^{n-\nu}s_{\nu},\quad n=0,1,2,\cdots,$$

where r is a constant which satisfies $0<r\leq 1$. The case $r=1$ corresponds to the ordinary convergence. The Lebesgue constants for this method are given by L. Lorch [1] for the case $r=\frac{1}{2}$, i.e.

$$L\left(n;\frac{1}{2}\right)=\frac{2}{\pi^2}\log 2n+A+o(1)\qquad \text{as } n\to\infty,$$

where

$$(1)\qquad A=-\frac{C}{\pi^2}+\frac{2}{\pi}\int_0^1\frac{\sin u}{u}du-\frac{2}{\pi}\int_0^\infty\left\{\frac{2}{\pi}-|\sin u|\right\}\frac{du}{u}$$

and C is the Euler-Mascheroni constant. For $0<r<1$ these constants are given by A. E. Livingston [2], i.e.

$$L(n;r)=\frac{2}{\pi^2}\log\frac{2nr}{1-r}+A+o(1)$$
$$=L\left(nr/(1-r);\frac{1}{2}\right),$$

where A is defined by (1).

Next the (γ, r) method of summation associates with a given sequence $\{s_n\}$ the means

$$\sigma^*_{n,r}=\sigma^*_n=\sum_{\nu=n}^{\infty}\binom{\nu}{n}r^{n+1}(1-r)^{\nu-n}s_{\nu},\quad n=0,1,2,\cdots,$$

where r is a constant which satisfies $0<r\leq 1$ [3]. Since the case $r=1$ corresponds to the ordinary convergence, we may suppose $0<r<1$. The object of the present note is to investigate the Lebesgue constants for (γ, r) method of Fourier series. We prove the following theorem.

Theorem. *The Lebesgue constants for (γ, r) method are given by*

$$L^*(n;r)=\frac{2}{\pi^2}\log\frac{2n}{1-r}+A+o(1)$$
$$=L\left(\frac{n}{1-r};\frac{1}{2}\right)+o(1)\qquad \textit{as } n\to\infty,$$

where A is defined by (1).