111. On Certain Triangulated Manifolds

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V. Rohlin and A. Schwarz [5] and R. Thom [7] defined the combinatorial Pontrjagin classes of triangulated manifolds and proved the existence of triangulated 8-dimensional manifolds which admit no differentiable structures compatible[1] with their given triangulations. A corresponding result for triangulated 16-dimensional manifolds was proved by K. Srinivasacharyulu [6]. The purpose of this note is to prove the corresponding theorems for the dimensions of the form $4k$ $(2\leqq k\leqq 14,\ k\neq 3)$.

In §1 certain triangulated $4k$-dimensional manifolds are constructed and studied. In §2 the theorem is proved.

Our method is quite analogous to that of R. Thom, and closely related with J. Milnor [4]. The word *n-manifold* will always be used for a compact oriented n-dimensional manifold without boundary. The word "differentiable" will be used to mean "differentiable of class C^∞".

1. Let us consider two differentiable mappings of spheres into rotation groups:

$$f_1: S^m \to SO(n+1), \quad f_2: S^n \to SO(m+1).$$

For these mappings Milnor [4] defined the differentiable $(m+n+1)$-manifold $M(f_1, f_2)$ with the following properties:

i) If the mapping f_1 carries S^m into the subgroup $SO(n)\subset SO(n+1)$, then $M(f_1, f_2)$ is a topological sphere.

ii) There exists a differentiable bounded manifold[2] W whose boundary is $M(f_1, f_2)$.

Hereafter we assume that

(*) if $m=n$, the mappings f_1, f_2 both carry S^m into the subgroup $SO(m)\subset SO(m+1)$.

Then $M(f_1, f_2)$ is always a topological $(m+n+1)$-sphere.[3] Furthermore, the differentiable $(m+n+1)$-manifold $M(f_1, f_2)$ has a C^∞-triangulation (L, g), and this C^∞-triangulation can be extended to a C^∞-triangulation (K, f) of the differentiable $(m+n+2)$-manifolds W. Then L is a combinatorial manifold and K is a combinatorial bounded manifold whose boundary is L (cf. Whitehead [8], Milnor [2]).

1) For the precise definition, see Whitehead [8], Milnor [2].

2) bounded manifold=variété à bord.

3) Cf. Milnor [4].