110. On the Boundedness of Solutions of Difference-Differential Equations

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Introduction. In their paper [1], R. Bellman and K. L. Cooke have defined a kernel function $K(t, s)$ which has been used to obtain several theorems concerning the stability and boundedness of solutions of difference-differential equations with perturbed terms.

In the present paper, we shall establish some theorems on the boundedness of solutions of difference-differential equations which are, in general, not linear.

1. For the sake of simplicity, we consider an equation

$$x'(t)=A(t)x(t)+B(t)x(t-1)+w(t) \quad (0\leqq t<\infty) \tag{1.1}$$

under the conditions

$$x(t-1)=\varphi(t) \quad (0\leqq t<1) \quad \text{and} \quad x(0)=x_0. \tag{1.2}$$

It is supposed that $A(t)$, $B(t)$, and $w(t)$ are continuous for $0\leqq t<\infty$, $\varphi(t)$ is continuous for $0\leqq t<1$, and $\lim_{t\to 1-0}\varphi(t)=\varphi(1-0)$ exists. Then, it is well known that there exists a unique solution of (1.1) under the initial conditions (1.2) for $0\leqq t<\infty$.

Now, we define a transformation

$$y(t)=\begin{cases} x(t)-\varphi(t+1) & (-1\leqq t<0), \\ x(t)-x_0 & (0\leqq t<\infty). \end{cases} \tag{1.3}$$

Then, by (1.3), (1.1) is reduced to the equation with respect to y, that is,

$$y'(t)=A(t)y(t)+B(t)y(t-1)+w_1(t) \tag{1.4}$$

under the condition $y(t-1)\equiv 0$ $(0\leqq t\leqq 1)$, where $w_1(t)$ is as follows:

$$w_1(t)=\begin{cases} x_0A(t)+B(t)\varphi(t)+w(t) & (0\leqq t<1), \\ x_0A(t)+x_0B(t)+w(t) & (1\leqq t<\infty). \end{cases}$$

By using the same kernel function $K(t, s)$ as defined in [1], the unique solution $y=y(t)$ of (1.4) under the condition $y(t-1)\equiv 0$ on $0\leqq t\leqq 1$ is represented by the integral

$$y(t)=\int_0^t K(t, s)w_1(s)ds \quad (0\leqq t<\infty).^{1)} \tag{1.5}$$

Thus, it follows from (1.3) that

$$x(t)=x_0+\int_0^t K(t, s)w_1(s)ds \quad (0\leqq t<\infty). \tag{1.6}$$

1) The method to obtain (1.5) is just the same as in [1].