137. A Remark on the Unique Continuation Theorem for Certain Fourth Order Elliptic Equations

By Taira SHIROTA
Osaka University

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1. Unique continuation theorems for solutions of certain fourth order elliptic equations, which are iterations of two second order elliptic equations, are considered by R. N. Pederson [4], S. Mizohata [3] and L. Hörmander [2].

Here we prove the following results with weaker vanishing requirements than these authors.

Theorem 1. *Let $L^{(i)}(x, D)$ $(i=1,2)$ be homogeneous, second order elliptic operators with coefficients of class C^2 in a neighbourhood G of the origin in Euclidean n-space such that $L^{(1)}(0,\xi)=L^{(2)}(0,\xi)$. Let $L(x,\xi)=L^{(1)}(x,\xi)L^{(2)}(x,\xi)$. If a function $u(x)$ of class C^4 in G satisfies the following two conditions*:

(1.1) *for any* $\alpha>0$

$$\lim_{r\to 0}\Big\{\sum_{|\beta|\leqq 4}|D^\beta u|\Big\}r^{-\alpha}=0,$$

(1.2) *for a positive number* M

$$|L(x,D)u(x)|^2\leqq M\Big\{|u(x)|^2r^{-6}+\sum_{|\beta|=1}|D^\beta u(x)|^2r^{-4}+\sum_{|\beta|=2}|D^\beta u(x)|^2r^{-2}+\sum_{|\beta|=3}|D^\beta u(x)|^2\Big\}\qquad (x\in G),$$

then $u(x)$ identically vanishes in a neighbourhood of the origin.

The proof is based on the method used by H. O. Cordes [1] and R. N. Pederson [4], but we use only the transformation $s=r\int_0^r(e^{-m_0\tau}-1)\frac{1}{\tau}d\tau$. The result was suggested by Professor H. Yamabe and Dr. S. Ito.

2. Let $K^{(m)}(R_1)$ be a class of functions $u(x)$ satisfying the following three conditions:

(2.1) $u(x)$ is defined in a cubic neighbourhood G of the origin with radius R and is in class $C^m(G)$, for any $\alpha>0$

$$\lim_{r\to 0}\Big\{\sum_{|\beta|\leqq m}|D^\beta u|\Big\}r^{-\alpha}=0, \tag{2.2}$$

(2.3) $u(x)=0$ for any x such that $|x|\geqq R_1$ $(R_1<R)$.

Lemma 1. *Let L be an elliptic operator of order 2 represented by polar coordinate systems such that*