136. Note on Metrizability and n-Dimensionality

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(Comm. by K. KUNUGI, M.J.A., Nov. 12, 1960)

This note gives firstly some metrizability conditions which are trivial corollaries of J. Nagata's general metrization theorem (§ 1). Our metrization theorem can be applied to criterions for n-dimensionality of metric spaces with the aid of the concept 'cushioned refinements' obtained by E. Michael (§ 2). One of the benefits of this interesting concept is to include both closed closure-preserving refinements and open star-refinements (cf. Remark 1.5). Thus our criterion for n-dimensionality provides us with more general form than [4, Theorem 7.2 and Theorem 7.5] where closed closure-preserving refinements and open star-refinements are essentially used respectively. It is to be noted that throughout this note a covering need not be open.

§ 1. **Metrizability.** **Lemma 1.1** (J. Nagata's general metrization theorem [5, Theorem 1]). *In order that a topological space R be metrizable it is necessary and sufficient that one can assign a neighborhood basis $\{U_i(x);\ i=1,2,\cdots\}$, neighborhood systems $\{S_i^1(x);\ i=1,2,\cdots\}$ and $\{S_i^2(x);\ i=1,2,\cdots\}$ satisfying the following conditions.*

(1) *$y\notin U_i(x)$ implies $S_i^2(y)\frown S_i^1(x)=\phi$(=the empty-set).*

(2) *$y\in S_i^1(x)$ implies $S_i^2(y)\subset U_i(x)$.*

Theorem 1.2. *In order that a topological space R be metrizable it is necessary and sufficient that there exists a sequence of coverings $\mathfrak{H}_i$, $i=1,2,\cdots$, of R which satisfies the following conditions.*

(3) *For any point x of R and any neighborhood U of x there exists i with $\mathrm{S}(x,\mathfrak{H}_i)$[1] $\subset U$.*

(4) *For any point x of R and any i there exists j with $x\notin \overline{\mathrm{S}(R-\mathrm{S}(x,\mathfrak{H}_i),\mathfrak{H}_j)}$.*

Proof. Since the necessity is clear, we prove only the sufficiency.

i) When $x\notin\overline{\mathrm{S}(R-\mathrm{S}(x,\mathfrak{H}_i),\mathfrak{H}_j)}$, let us put

$$U_{ij}(x)=\mathrm{S}(x,\mathfrak{H}_i),$$
$$S_{ij}^1(x)=R-\mathrm{S}(R-\mathrm{S}(x,\mathfrak{H}_i),\mathfrak{H}_j),$$
$$S_{ij}^2(x)=\mathrm{S}(x,\mathfrak{H}_j).$$

ii) When $x\in\overline{\mathrm{S}(R-\mathrm{S}(x,\mathfrak{H}_i),\mathfrak{H}_j)}$, let us put

$$U_{ij}(x)=S_{ij}^1(x)=R,$$
$$S_{ij}^2(x)=\mathrm{S}(x,\mathfrak{H}_i).$$

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1) $\mathrm{S}(x,\mathfrak{H}_i)=\smile\{H;\ x\in H\in\mathfrak{H}_i\}$. When R_1 is a subset of R, $\mathrm{S}(R_1,\mathfrak{H}_i)=\smile\{H;\ R_1\frown H\neq\phi,\ H\in\mathfrak{H}_i\}$.