135. On the Dimension of Product Spaces

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The purpose of the present note is to give a sufficient condition under which the inequality $\operatorname{Ind} R\times S\leq \operatorname{Ind} R+\operatorname{Ind} S$ holds good, where Ind denotes the large inductive dimension. We define inductively $\operatorname{Ind} R$. Let $\operatorname{Ind}\phi=-1$, where ϕ is the empty set. $\operatorname{Ind} R\leq n\ (=0,1,2,\cdots)$ if and only if for any pair $F\subset G$ of a closed set F and an open set G there exists an open set H with $F\subset H\subset G$ such that $\operatorname{Ind}(\bar{H}-H)\leq n-1$. When $\operatorname{Ind} R\leq n-1$ is false and $\operatorname{Ind} R\leq n$ is true, we call $\operatorname{Ind} R=n$. When $\operatorname{Ind} R\leq n$ is false for any n, we call $\operatorname{Ind} R=\infty$.

Let $\mathfrak{U}$ be a collection of subsets of a topological space R. Then we call $\mathfrak{U}$ is *discrete* or *locally finite* if every point of R has a neighborhood which meets at most respectively one element or finite elements of $\mathfrak{U}$. We call $\mathfrak{U}$ is *σ-discrete* or *σ-locally finite* if $\mathfrak{U}$ is a sum of a countable number of discrete or locally finite subcollections respectively. A *binary covering* is a covering which consists of two elements.

Lemma 1. *Let R be a hereditarily paracompact Hausdorff space. Then the following statements are valid.*

1) (Subset theorem). *For any subset T of R* $\operatorname{Ind} T\leq \operatorname{Ind} R$.

2) (Sum theorem). *If F_i, $i=1,2,\cdots$, are closed,* $\operatorname{Ind}\bigcup_{i=1}^{\infty} F_i=\sup \operatorname{Ind} F_i$.

3) (Local dimension theorem). *For any collection $\mathfrak{U}$ of open sets* $\operatorname{Ind}\bigcup\{U;\ U\in\mathfrak{U}\}=\sup\{\operatorname{Ind} U;\ U\in\mathfrak{U}\}$.

This is proved by C. H. Dowker [1]. The main part of the following lemma is essentially proved in Morita [4], but we give here full proof for the sake of completeness.

Lemma 2. *In a hereditarily paracompact Hausdorff space R the following conditions are equivalent.*

1) $\operatorname{Ind} R\leq n$.

2) *Every open covering can be refined by a locally finite and σ-discrete open covering* $\mathfrak{V}$ such that for any $V\in\mathfrak{V}$ $\operatorname{Ind}(\bar{V}-V)\leq n-1$.

3) *Every binary open covering can be refined by a σ-locally finite open covering $\mathfrak{V}$ such that for any $V\in\mathfrak{V}$* $\operatorname{Ind}(\bar{V}-V)\leq n-1$.

Proof. First we prove the implication 1)$\rightarrow$2). Let $\mathfrak{U}$ be an arbitrary open covering of R; then by A. H. Stone's theorem [5] $\mathfrak{U}$