134. Some Topological Properties on Royden's Compactification of a Riemann Surface

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(Comm. by K. KUNUGI, M.J.A., Nov. 12, 1960)

1. Let R be an open Riemann surface and $M(R)$ be the totality of bounded a.c.T. functions on R with finite Dirichlet integrals and $M_0(R)$ be the totality of functions in $M(R)$ with compact supports. We denote by $M_\Delta(R)$ the closure of $M_0(R)$ in BD-convergence topology, where a sequence $\{\varphi_\nu\}$ converges to φ in BD-convergence topology if the sequence $\{\varphi_\nu\}$ is bounded and converges to φ uniformly on each compact subset of R and the sequence $\left\{\iint_R d(\varphi_\nu-\varphi)\wedge \overline{{}^*d(\varphi_\nu-\varphi)}\right\}$ converges to zero.

Royden's compactification R^* of R is the unique compact Hausdorff space containing R as its open and dense topological subspace such that any function in $M(R)$ can be uniquely extended to R^* so as to be continuous on R^*. The (*Royden's*) *ideal boundary* of R is defined by R^*-R and denoted by ∂R. The compact set $\Delta=\{p\in R^*;\ f(p)=0$ for all f in $M_\Delta(R)\}$ is a part of ∂R and called the *harmonic boundary* of R. We also say that $\partial R-\Delta$ is the non-harmonic boundary of R. These notions are introduced by Royden [3]. Our formulation above mentioned is different from that in [3] but equivalent to that of Royden. Details are in [1].

In this note we state some topological properties of R^* and solve a question raised in [3].

2. Consider a normal exhaustion $\{R_n\}_1^\infty$ of R in the sense of Pfluger [2]. The open set $R-\overline{R}_n$ is decomposed into a finite number of non-compact connected components $K_1^{(n)}, K_2^{(n)},\cdots,K_{N(n)}^{(n)}$. A determining sequence is a sequence $\{K_{i_n}^{(n)}\}_1^\infty$ such that

$$K_{i_1}^{(1)}\supset K_{i_2}^{(2)}\supset\cdots\supset K_{i_n}^{(n)}\supset K_{i_{n+1}}^{(n+1)}\supset\cdots\ . \qquad (1)$$

If we fix an exhaustion $\{R_n\}_1^\infty$, then the totality of determining sequences corresponds in a one-to-one and onto manner to the totality of *ends* of R in the sense of Kerékjártó-Stoïlow [2]. Let $\{E_k\}$ be the decomposition of ∂R into connected components. First we show

Theorem 1. *The decomposition $\{E_k\}$ can be regarded as the totality of ends of R in the sense of Kerékjártó-Stoïlow.*

Proof. An end is determined by a sequence (1). Then the intersection $\alpha=\bigcap_1^\infty \overline{K}_{i_n}^{(n)}$ is a continuum in ∂R, since each $\overline{K}_{i_n}^{(n)}$ is a con-