133. General Analyses of the Forced Oscillations in a Waveguide and in a Cavity

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The electromagnetic fields $\boldsymbol{E}$ and $\boldsymbol{H}$ in a homogeneous and isotropic medium with medium constants ε, μ and σ, satisfy the equations

$$(1) \qquad \nabla\times\boldsymbol{E}=-i\omega\mu\boldsymbol{H}-\boldsymbol{k}_H, \quad \nabla\times\boldsymbol{H}=(\sigma+i\omega\varepsilon)\boldsymbol{E}+\boldsymbol{k}_E$$

when known distributions of densities of electromagnetic currents $\boldsymbol{k}_E$ and $\boldsymbol{k}_H$ are given. In the other paper,[1] the author has shown that the fields in anisotropic inhomogeneous medium, when discussed along the line of the theory of perturbation, satisfy the same equations as (1), in which $\boldsymbol{k}_E$ and $\boldsymbol{k}_H$ have been the perturbed terms. Therefore the analysis of (1) is not only interesting in itself but useful for studies of the fields in anisotropic inhomogeneous medium.

1. Suppose that $\boldsymbol{i}_z$ is the unit vector along z-axis, and that $\boldsymbol{E}=\boldsymbol{E}_t+\boldsymbol{i}_zE_z$, $\boldsymbol{H}=\boldsymbol{H}_t+\boldsymbol{i}_zH_z$, $\boldsymbol{k}_E=\boldsymbol{k}_{Et}+\boldsymbol{i}_zk_{Ez}$, $\boldsymbol{k}_H=\boldsymbol{k}_{Ht}+\boldsymbol{i}_zk_{Hz}$ and $\nabla=\nabla_t+\boldsymbol{i}_z\,\partial/\partial z$, then (1) will be reduced to

$$(2) \qquad \partial\boldsymbol{E}_t/\partial z=\nabla_tE_z+i\omega\mu\boldsymbol{i}_z\times\boldsymbol{H}_t+\boldsymbol{i}_z\times\boldsymbol{k}_{Ht}, \quad i\omega\mu H_z=\nabla_t\cdot[\boldsymbol{i}_z\times\boldsymbol{E}_t]-k_{Hz}$$
$$\partial\boldsymbol{H}_t/\partial z=\nabla_tH_z-(\sigma+i\omega\varepsilon)\boldsymbol{i}_z\times\boldsymbol{E}_t-\boldsymbol{i}_z\times\boldsymbol{k}_{Et}, \quad (\sigma+i\omega\varepsilon)E_z=-\nabla_t\cdot[\boldsymbol{i}_z\times\boldsymbol{H}_t]-k_{Ez}.$$

To begin with, we shall consider about the fields in a waveguide, the axis of which is parallel to z axis. Let $\mathfrak{L}$ be the two sided Laplace transformation defined as $\mathfrak{L}\{\boldsymbol{F}(z)\}=\boldsymbol{F}(s)=s\int_{-\infty}^{\infty}e^{-sz}\boldsymbol{F}(z)dz$. Applying $\mathfrak{L}$ to (2), we shall have, after some calculations,

$$(3) \qquad \begin{aligned} k^2\boldsymbol{E}_t&=s\nabla_tE_z+i\omega\mu\boldsymbol{i}_z\times\nabla_tH_z+i\omega\mu\boldsymbol{k}_{Et}+s\boldsymbol{i}_z\times\boldsymbol{k}_{Ht} \\ k^2\boldsymbol{H}_t&=s\nabla_tH_z-(\sigma+i\omega\varepsilon)\boldsymbol{i}_z\times\nabla_tE_z+(\sigma+i\omega\varepsilon)\boldsymbol{k}_{Ht}-s\boldsymbol{i}_z\times\boldsymbol{k}_{Et} \end{aligned}$$

$$(4) \qquad \Delta_tE_z+k^2E_z=g_E, \quad \Delta_tH_z+k^2H_z=g_H,$$

where $\kappa^2=-i\omega\mu(\sigma+i\omega\varepsilon)$, $k^2=\kappa^2+s^2$, $\Delta_t=\nabla_t\cdot\nabla_t$ and $g_E=-s\nabla_t\cdot\boldsymbol{k}_{Et}/(\sigma+i\omega\varepsilon)-\nabla_t\cdot[\boldsymbol{i}_z\times\boldsymbol{k}_{Ht}]-k^2k_{Ez}/(\sigma+i\omega\varepsilon)$, $g_H=-s\nabla_t\cdot\boldsymbol{k}_{Ht}/i\omega\mu+\nabla_t\cdot[\boldsymbol{i}_z\times\boldsymbol{k}_{Et}]-k^2k_{Hz}/i\omega\mu$. Conversely, assume that E_z and H_z satisfy (4), and that $\boldsymbol{E}_t$ and $\boldsymbol{H}_t$ be defined by the right hand sides of (3) with the solutions E_z and H_z of (4), then it is easy to see that these values of $\boldsymbol{E}$ and $\boldsymbol{H}$ satisfy (2), that is (1). Therefore (1) is equivalent to (3) and (4).

Next we shall consider the fields in a cavity. Here a cavity means a finite domain which is enclosed by a waveguide and two planes of conductor perpendicular to the axis of the waveguide. Let these planes be $z=0$ and $z=L$. Suppose that the finite Fourier sine and cosine transformations of $\theta(z)$, which is a function defined in