132. *Perturbation Theory of the Electromagnetic Fields in Anisotropic Inhomogeneous Media*

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(Comm. by K. KUNUGI, M.J.A., Nov. 12, 1960)

Since it is very difficult and complicated to analyse the fields in anisotropic inhomogeneous media exactly, some approximated methods have been studied, but it seems to the author that few of them are rigorous and general enough. In this paper he will develop the perturbation theory of the fields which is not only simple but rigorous and general under one assumption that the deviations of the anisotropicity and inhomogeneity are not so large.

1. Suppose that the properties of the medium are represented as $\boldsymbol{D}=[\varepsilon]\boldsymbol{E}+[\xi]\boldsymbol{H}$, $\boldsymbol{B}=[\mu]\boldsymbol{H}+[\zeta]\boldsymbol{E}$ and $\boldsymbol{K}=[\sigma]\boldsymbol{E}$, where $[\varepsilon]$ etc. are 3×3 matrices, the elements ε_{ij} $(i, j=1, 2, 3)$ of which are functions of position. Let $\varepsilon'_{ij}=\varepsilon_{ij}-\varepsilon\delta_{ij}$ where ε is a properly chosen constant, then $[\varepsilon]=\varepsilon U+[\varepsilon']$, where U is the unit matrix and $[\varepsilon']$ is a matrix with elements ε'_{ij}. Similar holds for $[\mu]$ and $[\sigma]$, and Maxwell's equation will be

$$(1)\qquad \nabla\times\boldsymbol{E}=-i\omega\mu\boldsymbol{H}-\boldsymbol{K}_H,\qquad \nabla\times\boldsymbol{H}=(\sigma+i\omega\varepsilon)\boldsymbol{E}+\boldsymbol{K}_E$$

where $\boldsymbol{K}_H=i\omega([\zeta]\boldsymbol{E}+[\mu']\boldsymbol{H})$ and $\boldsymbol{K}_E=[\sigma'+i\omega\varepsilon']\boldsymbol{E}+i\omega[\xi]\boldsymbol{H}$. (1) represents the fields in isotropic and homogeneous medium with medium constants ε, μ and σ, in the presence of the distributions of densities of electromagnetic currents $\boldsymbol{K}_E$ and $\boldsymbol{K}_H$.

Assume that $\bar{\varepsilon}=\dfrac{1}{3}\sum_n \varepsilon_{nn}$ and $\varepsilon=\int\bar{\varepsilon}dV\Big/\int dV$, i.e. $\bar{\varepsilon}$ is the mean value of the diagonal elements of $[\varepsilon]$, which may still be a function of position, and ε is a constant which is the mean value of $\bar{\varepsilon}$ in the domain with which we are concerned. Constants μ and σ will be obtained in a similar way. With these constants we shall assume that

$$(2)\qquad \left|\frac{\varepsilon'_{mn}}{\varepsilon}\right|,\ \left|\frac{\xi_{mn}}{\varepsilon}\right|,\ \left|\frac{\mu'_{mn}}{\mu}\right|,\ \left|\frac{\zeta_{mn}}{\mu}\right|,\ \left|\frac{\sigma'_{mn}}{\sigma}\right|=0(\epsilon)$$

where ϵ is a number less than 1 and $0(\epsilon)$ is the Landau symbol. Under this assumption, it is easy to see that $\boldsymbol{K}_E$ and $\boldsymbol{K}_H$ are quantities of $0(\epsilon)$, and that when $\epsilon\to 0$, (1) reduces to

$$(3)\qquad \nabla\times\boldsymbol{E}_0=-i\omega\mu\boldsymbol{H}_0,\qquad \nabla\times\boldsymbol{H}_0=(\sigma+i\omega\varepsilon)\boldsymbol{E}_0$$

where $\boldsymbol{E}_0$ and $\boldsymbol{H}_0$ are the fields in isotropic and homogeneous medium with medium constants ε, μ and σ. Therefore if we put as $\boldsymbol{E}=\boldsymbol{E}_0+\boldsymbol{e}$,