131. On Quasi-normed Spaces over Fields with Non-archimedean Valuation

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The normed spaces over the fields with non-archimedean valuation were established by A. F. Monna [1]. In this paper, we shall consider the quasi-normed spaces over the fields with non-archimedean valuation.

Let K be a complete field with a non-archimedean valuation $|\lambda|$. We shall fix this field K throughout this paper.

1. General properties. **Definition 1.1.** *Let E be a linear space over a field K. An application $\|x\|$ of x is called a non-archimedean (n.a.) quasi-norm with the power r if it satisfies the following axioms:*

1. $\|x\|=0$ *if and only if* $x=\theta$.
2. $\|x+y\|\leq\max(\|x\|, \|y\|)$ *for all* $x, y\in E$.
3. $\|\lambda x\|=|\lambda|^r\|x\|$ *for* $\lambda\in K$ *and* $x\in E$, (r *real* $0<r<\infty$).

Let $\|x\|$ be a n.a. quasi-norm with the power r and let $d(x, y)=\|x-y\|$, $x\in E$, $y\in E$ then d is the distance on E. A linear topological space which is defined by the distance d is called a n.a. quasi-normed space with the power r.

Definition 1.2. *Let E be a n.a. quasi-normed space with the power r and if E is complete with the distance d, E will be called a n.a. (QN) space with the power r.*

We can prove the usual properties of quasi-normed spaces in n.a. quasi-normed spaces by the same ways [2-4] and [5]. Therefore we have the following theorems.

Theorem 1.1. *Let E be a n.a. (QN) space with the power r and N a closed subspace, then the quotient space E/N is a n.a. (QN) space with the power r.*

Theorem 1.2. *If E is a n.a. quasi-normed space with the power r then the space may be regarded as a dense subspace of a n.a. (QN) space $\widehat{E}$ with the power r.*

We omit the proofs of the general theorems since they are proved by the same way as the archimedean case.

2. Linear transformations. Let E, F be two n.a. quasi-normed spaces with powers r, s and T a linear transformation which maps E into F.

Theorem 2.1. *A linear transformation T is continuous if and only if there exists a positive number M for which the following inequality holds:*