128. *Integral Transforms and Self-dual Topological Rings*

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It is well known that a generalization of the Poisson summation formula holds on some types of topological groups [1, 3]. In this paper we shall show that if the Poisson summation formula holds in some sense on a locally compact topological ring then the ring is self-dual as an additive group (Proposition 2). In this paper we shall use the following notations:

R is a locally compact ring with a neutral element 1,

R^+ is the additive group composed of all elements of R,

$\hat{R}$ is the dual group of R^+,

μ is a Haar-measure on R^+.

To any measurable functions $f(x)$, $g(x)$, $T(x)$ defined on R $f*g$ is the convolution of f and g on R^+,

Car(f) is the carrier of f and

$$Tf(x)=\int_R T(xy)f(y)d\mu(y).$$

Finally $\mathfrak{D}^0$ is the set of all continuous functions with compact carrier defined on R.

§1. **Proposition 1.** *Let $T(x)$ be a bounded continuous function on G but be not constant* 0. *If*

(1) $$T(f*g)=Tf\cdot Tg \qquad \text{for all } f, g\in\mathfrak{D}^0,$$

then $T\in\hat{R}$.

Proof. Let us denote $f_u(x)=f(x+u)$ and $P_f(-u)=\dfrac{Tf_u(1)}{Tf(1)}$. (Naturally P_f is defined to f such that $Tf(1)\neq 0$.) By the hypothesis and the definition of the convolution we have

$$Tf_u\cdot Tg=T(f_u*g)=T(f*g_u)=Tf\cdot Tg_u,$$

and then $P_f(-u)=P_g(-u)$. Therefore we shall denote simply $P(-u)$.

Concerning the function $P(u)$ we get

(2) $$P(u+v)=P(u)P(v),$$

for $P(-u-v)=\dfrac{T(f*f)_{u+v}(1)}{T(f*f)(1)}=\dfrac{T(f_u*f_v)(1)}{T(f*f)(1)}=\dfrac{Tf_u(1)\cdot Tf_v(1)}{Tf(1)\cdot Tf(1)}=P(-u)P(-v).$

For any positive number ε and any $f\in\mathfrak{D}^0$ there exists an open set of R such that

$$|Tf_u(1)-Tf(1)|\leqq\int_R |T(x)||f(x+u)-f(x)|\,d\mu(x)<\varepsilon$$