7. *On Transformation of the Seifert Invariants*

By Joseph WEIER

(Comm. by K. KUNUGI, M.J.A., Jan. 12, 1961)

The theory of continuous transformations of manifolds shows preference to the case that $\dim X = \dim Y$ or $\dim X > \dim Y$ where X is mapped into Y. The reason is that every continuous mapping of an m-sphere into an n-sphere with $m<n$ is homotopic to zero. We will cast a look on the case $\dim X < \dim Y$.

1. Suppose z, z' are two disjoint zero-divisors in the compact manifold X such that $\dim z + \dim z' \geq (\dim X) - 1$. Then the pair (z, z') determines [1] a rational interlacing cycle, $\sigma(z, z')$, as follows. Let a, b be the smallest positive integers satisfying $az \sim 0$ and $bz' \sim 0$, and let A, B be two finite integral chains in X such that $\partial A = az$ and $\partial B = bz'$. Then, if f denotes the usual intersection function,

$$\frac{1}{a}f(A, z') = \frac{1}{ab}f(A, \partial B) = \pm\frac{1}{ab}f(\partial A, B) = \pm\frac{1}{ab}f(az, B) = \pm\frac{1}{b}f(z, B).$$

One thus obtains an expression that does not depend on A. Now

$$\sigma(z, z') = \frac{1}{a}f(A, z')$$

is Seifert's interlacing cycle.

2. Let $2 \leq m < n$ be integers, let M be an m-dimensional and N an n-dimensional oriented differentiable compact manifold, moreover $f: M \to N$ a continuous mapping. Let P, Q, R, S be pairwise disjoint oriented differentiable compact manifolds in N such that

$$p \geq n-m, \quad q \geq n-m, \quad r \geq n-m, \quad s \geq n-m,$$
$$p+q+r+s = 4n-m-3, \qquad p+q \geq 2n-m,$$

where p, q, r, s are the dimensions of P, Q, R, S respectively. For instance setting

$$p = q = r = n-1 \quad \text{and} \quad s = n-m,$$

one confirms at once that the above dimensional suppositions are fulfilled.

The algebraic inverse of P, Q, R, S under f, defined for instance in [4], will be denoted by z_P, z_Q, z_R, z_S respectively. Geometrically one can suppose [5] that the inverses of P, Q, R, S are differentiable manifolds. Then z_P, z_Q, z_R, z_S is an integral cycle of dimension $p-(n-m)$, $q-(n-m)$, $r-(n-m)$, and $s-(n-m)$ respectively. Let the manifolds P, Q, R, S be defined in such a way that z_P, z_Q, z_R, z_S are zero-divisors. That is always possible as one easily confirms. Let z_T denote the above defined Seifert interlacing cycle, $\sigma(z_P, z_Q)$. By

$$\begin{aligned}\dim z_T &= (\dim z_P) + 1 + \dim z_Q - \dim M \\ &= (p-n+m)+1+(q-n+m)-m = p+q-2n+m+1\end{aligned}$$