3. *Uniform Extension of Uniformly Continuous Functions*

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In this note, a space is uniform and a function is, unless otherwise specified, real valued and uniformly continuous.

Katětov proved [3, Theorem 3] that, if A is an arbitrary uniform subspace of a space S, then any bounded function on A can be uniformly extended to S. In this note, we are going to find conditions under which the same kind of extension holds for not necessarily bounded functions. In other words, when we say that a space S has *a property* E if any function on an arbitrary uniform subspace of S can be uniformly extended to S, then we shall see in the following some conditions of S in order to have the property E. A space is said to be *uc* if every real valued continuous function on the space is uniformly continuous. Some characterisations for a space to be *uc* are known [1]. When S is normal and *uc*, then S has the property E, this is a trivial sufficient condition. Another sufficient condition is well known [2, Theorem 4.12], which is however not necessary even in a metric space. Theorem 2 gives a necessary and sufficient condition in a pseudo-metric space, and it also induces a necessary and sufficient condition of a space to have a restricted property E.

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The following theorem is a corollary to the Katětov's theorem [3, Theorem 3], which however gives a sufficient condition for the property E which does not induce the local fineness [2].

Theorem 1. *Let* $\{f^\alpha\}$ *be a uniformly equicontinuous family of functions* f^α *on a uniform subspace* A *of a space* S *into closed intervals* $[a^\alpha, b^\alpha]$, $0<b^\alpha-a^\alpha=c^\alpha<c<\infty$, *then there is a uniformly equicontinuous family of uniform extensions of* f^α *to* S.

Proof. Let S' be the union of disjoint copies S^α of S for all α, then the family of unions V'_β of disjoint copies V^α_β of all entourages V_β in S generates a uniform structure in S', and f defined by $f^\alpha-c^\alpha$ is uniformly continuous on $\smile A^\alpha, A^\alpha$ copies of A, to $[0,c]$. By the Katětov's theorem, there is a uniform extension g of f to S'. $g^\alpha+c^\alpha$ is desired extension of f^α.

We can prove, in an elementary way similar to the well-known proof of the Urysohn's extension theorem in normal spaces, that the