1. Note on Paracompactness

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1. Suggested by a well-known theorem of C. H. Dowker [1] that a topological space is countably paracompact and normal if and only if the product space $X\times I$ is normal, we have established the following theorem in a previous paper [2].

Theorem 1.1. *A topological space X is $\mathfrak{m}$-paracompact and normal if and only if the product space $X\times I^{\mathfrak{m}}$ is normal, where $\mathfrak{m}$ is an infinite cardinal number.*

Here a topological space X is called $\mathfrak{m}$-paracompact if any open covering of power$\leqq\mathfrak{m}$ admits a locally finite open refinement, and $I^{\mathfrak{m}}$ means the product space of $\mathfrak{m}$ copies of I, where $\mathfrak{m}$ is a cardinal number and I is the closed line interval [0, 1]. A topological space X is, by definition, paracompact if X is $\mathfrak{m}$-paracompact for any cardinal number $\mathfrak{m}$; furthermore, X is paracompact if X is $\mathfrak{m}$-paracompact for a cardinal number $\mathfrak{m}$ not less than the power of an open base of X. Accordingly, Theorem 1.1 gives a new characterization of paracompact spaces. Of course, "$\aleph_0$-paracompact" is nothing else "countably paracompact".

The purpose of this paper is to prove the following theorem which is a generalization of Theorem 1.1.

Theorem 1.2. *A topological space X is $\mathfrak{m}$-paracompact and normal if and only if the product space $X\times C^{\mathfrak{m}}$ is normal, where C is any compact metric space containing at least two points and $C^{\mathfrak{m}}$ means the product space of $\mathfrak{m}$ copies of C, and $\mathfrak{m}$ is an infinite cardinal number.*

As a special case where C is a space consisting of exactly two points we obtain the following theorem.

Theorem 1.3. *A topological space X is $\mathfrak{m}$-paracompact and normal if and only if the product space $X\times D^{\mathfrak{m}}$ is normal, where D is a discrete space consisting of two points and $D^{\mathfrak{m}}$ means the product space of $\mathfrak{m}$ copies of D, and $\mathfrak{m}$ is a cardinal number$\geqq 1$.*

The space $D^{\mathfrak{m}}$ is called a Cantor space, and $D^{\aleph_0}$ is the Cantor discontinuum.

It should be noted that in case $\mathfrak{m}=\aleph_0$, as far as the "if" part is concerned Theorem 1.3 gives a stronger form than Dowker's theorem while Theorem 1.1 gives a weaker form, and that for a finite cardinal number $\mathfrak{m}\geqq 1$, Theorem 1.3 is true but Theorem 1.1 is not.