24. On Distribution Solution of Partial Differential Equations of Evolution. II

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We shall continue the study of the properties of the classes $\mathfrak{E}_t^s\mathfrak{D}'$ in section 3 and prove the main theorem 6 in section 4.

3. THEOREM 3. *Let $(G_n)_0$ be a subdomain of G_n and $a\leqq a_0<b_0\leqq b$ and let $\tilde{T}\in\mathfrak{E}_t^s\mathfrak{D}'(G_{n+1})$ $(+\infty\geqq s>-\infty)$. Then the restriction $(\tilde{T})_0$ of $\tilde{T}$ on $(G_{n+1})_0(=(G_n)_0\times(a_0,b_0))$ belongs to $\mathfrak{E}_t^s\mathfrak{D}'[(G_{n+1})_0]$.*

PROOF. The proof follows immediately from the definitions of the classes $\mathfrak{E}_t^s\mathfrak{D}'$, so we omit the proof of Theorem 3.

THEOREM 4. *Let $(G_n)_0$ be a domain in R^n such that $\overline{(G_n)_0}\subseteq G_n$ and $\overline{(G_n)_0}$ is compact. Also let $-\infty\leqq a<a_0<b_0<b\leqq+\infty$. If $\tilde{T}\in\mathfrak{D}'(G_{n+1})$, then there is an integer s such that the restriction $(\tilde{T})_0$ of $\tilde{T}$ on $(G_{n+1})_0(=(G_n)_0\times(a_0,b_0))$ belongs to $\mathfrak{E}_t^s\mathfrak{D}'[(G_{n+1})_0]$.*

PROOF. By the local structure theorem of distributions,[1] we can find a complex-valued function $F_0\in C^0[(G_{n+1})_0]$ such that $(\tilde{T})_0=D_t^{s'}D_x^{\alpha}F_0$ s' an integer$\geqq 0$. By Lemma 2, F_0 regarded as a distribution belongs to $\mathfrak{E}_t^0\mathfrak{D}'[(G_{n+1})_0]$. Hence by (2.6) in Theorem 2 and by Theorem 1, we have $(\tilde{T})_0\in\mathfrak{E}_t^s\mathfrak{D}'[(G_{n+1})_0]$ $s=-s'$. Q.E.D.

THEOREM 5. *Let $\tilde{T}\in\mathfrak{D}'(G_{n+1})$. Assume that each point (x_0,t_0) of G_{n+1} has a neighbourhood $(G_{n+1})_0$ of the form $(G_n)_0\times(a_0,b_0)$ where $-\infty\leqq a\leqq a_0<b_0\leqq b\leqq+\infty$ and $(G_n)_0$ is a subdomain of G_n such that the restriction $(\tilde{T})_0$ of $\tilde{T}$ on $(G_{n+1})_0$ belongs to $\mathfrak{E}_t^s\mathfrak{D}'[(G_{n+1})_0]$ where s $(-\infty<s\leqq+\infty)$ is the same for all points $(x_0,t_0)\in G_{n+1}$. Then $\tilde{T}\in\mathfrak{E}_t^s\mathfrak{D}'(G_{n+1})$.*

PROOF. For $+\infty\geqq s\geqq 0$, the proof of Theorem 5 is immediate if a suitable partition of the unity[2] on G_{n+1}, the univalency of the mapping M^{-1} and the compactness of the carriers of the test functions φ for the distribution $\tilde{T}$ are used. Hence we omit the proof for the case.

For $-\infty<s<0$, we proceed as follows. For $\tilde{T}\in\mathfrak{D}'(G_{n+1})$, there exists always a distribution $\tilde{T}_3\in\mathfrak{D}'(G_{n+1})$ such that $\tilde{T}=D_t^{-s}\tilde{T}_3$.[3] Then

1) Cf. L. Schwartz [2], p. 83.
2) Cf. L. Schwartz [2], p. 23.
3) Cf. L. Schwartz [2], p. 55. The same remark as in 7) applies here also.