23. *On Distribution Solution of Partial Differential Equations of Evolution. I*

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1. **Introduction.** When the solutions $u_i(x, t)$ of a system of linear partial differential equations of evolution

(1.1) $\quad D_t u_i = \sum_{|\alpha| \leq l} \sum_{j=1}^{n} a_{i,j,\alpha}(x, t) D_x^{\alpha} u_j + b_i(x, t) \quad i=1,\cdots,n$

(α_k ($k=1,\cdots,n$) non-negative integers $\alpha=(\alpha_1,\cdots,\alpha_n)$ $|\alpha|=\sum_{k=1}^{n}\alpha_k$ $x=(x_1,\cdots,x_n)$ $D_x^{\alpha}=D_{x_1}^{\alpha_1}\cdots D_{x_n}^{\alpha_n}$ D_{x_i}, D_t: the operators of partial differentiation with respect to x_i and to t, and l: a non-negative integer) are discussed, $u_i(x, t)$ are sometimes[1] considered as continuously differentiable functions of t whose values are distributions in (x)-space in the sense of L. Schwartz. But coordinate transformations mixing the space coordinates x_i and the time coordinate t are important for some problems. For such problems, solutions in different coordinate systems are compared most naturally by considering them as distributions in (x, t)-space in the sense of L. Schwartz. Not only for such reasons but also by itself, it is of some interest to ask: when can a distribution solution u_i ($i=1,\cdots,n$) in (x, t)-space of a system of equations of evolution (1.1) where $a_{i,j,\alpha}(x, t)$ are infinitely differentiable functions of (x, t) and $b_i(x, t)$ are distributions in (x, t)-space be considered as a set of continuously differentiable functions of t whose values are distributions in (x)-space?

The main theorem 6 in section 4 of this note shows that this is the case, if and only if in (1.1) $b_i(x, t)$ are distributions in (x, t)-space which can be considered as continuous functions of t whose values are distributions in (x)-space.[2] Theorem 6 contains also more precise results. If $a_{i,j,\alpha}(x, t)$ are infinitely differentiable, all distributions $u_i(x, t)$ in (x, t)-space constituting a solution of (1.1) belong to a class $\mathfrak{E}_t^{s+1}\mathfrak{D}'$ generally by one step more regular with respect to t than a class $\mathfrak{E}_t^{s}\mathfrak{D}'$ $(+\infty \geqq s > -\infty)$ (but $\mathfrak{E}_t^{s+1}\mathfrak{D}'=\mathfrak{E}_t^{s}\mathfrak{D}'$, if $s=+\infty$) to which all distributions $b_i(x, t)$ in (x, t)-space in the right sides of (1.1) belong. Cf. Definitions 3 and 5. Also every distribution in (x, t)-space belongs locally to a class $\mathfrak{E}_t^{s}\mathfrak{D}'$ $(+\infty \geqq s > -\infty)$ by Theorem 4.

As preparations to section 4, we shall classify distributions in (x, t)-space according to their regularity with respect to t and prove related theorems in sections 2 and 3.

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1) For example, L. Schwartz [1].

2) In section 4, we shall give a precise formulation of the above statements.