34. A Certain Type of Vector Field. II

By Toshiyuki MAEBASHI

Department of Mathematics, Hokkaido University

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All the notations of the previous paper [2] are included in the present paper.

I. Let C be a circle and let π be a symmetry, i.e. an idempotent isometry ($\neq$ the identity) leaving $O\in C$ fixed. Now consider an imbedding I of C into a 2-dimensional Euclid subspace E_2 of an n-dimensional Euclid space E_n. If O' is the other fixed point of π, denote by G the totality of Euclid motions g of E_n leaving both of $I(O)$ and $I(O')$ fixed. Then the orbit S by G of $I(C)$ is referred to as a *compact space of rotation*, if π keeps the curvature of the curve $I(C)$ invariant. Given a function $f(s)$ on C with $f\circ\pi(s)=f(s)$, we can extend it to one defined on the whole S in this way: Let $x\in S$ and $x=g(I(s))$ for $g\in G$ and $s\in C$. Then we set $f(s)=f(x)$. It is easy to see that $f(x)$ is well-defined.

Let $f_1(s)$ be such a function that $df_1\neq 0$ except at O and O' and the above condition $f_1\circ\pi=f_1$ hold. Then as is easily seen, the dual vector V_1 of *Grad* $[f_1(x)]$ satisfies 2) and 3) of Theorem A. Furthermore for s such that $V_1(I(s))$ is not proportional to $V_1(I\circ\pi(s))$, the vector field V satisfies 1) also at $x=g(I(s))$ for every $g\in G$. In fact there is a function f_2, in the neighborhood of such s, of the nature that the end point of the vector V_2[1] dual to *Grad* $[f_2(x)]$ remains fixed for the movement of $x\in S$ stated in the theorem. In addition, we can suppose that $f_2(s)$ has been chosen in such a fashion that π leaves $f_2(s)$ invariant and the domain of $f_2(s)$ is the set of all the s of the above-prescribed nature. For simplicity let us assume that the exceptional s are nowhere dense. Then V_1 and V_2 have the following properties respectively (we see these from Theorem A).

(1) *V_1 is a differentiable vector field defined on the whole S.*

(2) *The dual 1-form ω_1 to V_1 is closed.*

(3) *$A_{V_1}\in\mathfrak{P}^{-1}(\mathfrak{S}^*)$[2] except at $I(O)$ and $I(O')$.*

(1*) *V_2 is a differentiable vector field defined on a dense open*

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1) Take a straight-line passing through $I(O)$ to the direction of $I(O')$ for the a-axis and introduce an orthogonal coordinate system in E_2. Then we have

$$\| V_2(I(s)) \| = \sqrt{1+\left(\frac{da}{db}\right)^2}\, b$$

where a and b are the coordinates of $I(s)$.

2) For an exceptional s, we have $\mathfrak{P}(A_{V_1})=0$ at $x=g\circ I(s)$ ($g\in G$).