32. A Note on Hausdorff Spaces with the Star-finite Property. I

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K. Morita [4, Theorem 10.3] proved that (α) every metric space with the star-finite property can be embedded into the product of a 0-dimensional metric space and the Hilbert fundamental cube.[1] Yu. M. Smirnov [5] proved the following theorem which is an immediate corollary of Morita's theorem cited now and seems to be probably obtained independently of Morita's work: (β) For every metric space R with the star-finite property there exist a 0-dimensional metric space S and a continuous mapping f of R onto S such that $f^{-1}(x)$ is separable for every point x of S.

The purpose of this note is to give an analogous proposition to (β) as follows.

Theorem 1. *Let R be a non-empty Hausdorff space with the star-finite property.*[2] *Then there exist a paracompact Hausdorff space A with $\dim A$*[3]*$=0$ and a continuous mapping f of R onto A such that for every point y of A $f^{-1}(y)$ has the Lindelöf property.*[4]

In view of Morita's theorem [2] we may expect that the condition imposed on f in our Theorem 1 will be strengthened to be closed: But it is, in general, impossible as Yu. Smirnov's example [5] shows. It seems to be difficult to obtain a refinement of Theorem 1 in an analogous expression to the proposition (α), because of the difficulty to get the space for our case which plays the same rôle as the Hilbert fundamental cube does for the metric spaces with the star-finite property.

To prove Theorem 1 let us start with finding the universal 0-dimensional paracompact Hausdorff spaces.

Definition. Let Λ be a directed set and $\{A_\lambda, f_{\lambda\mu}; \mu<\lambda, \mu, \lambda\in\Lambda\}$ be an inverse limiting system consisting of discrete spaces A_λ, where

1) This theorem has been improved by himself as follows: Every metric space with an open basis which is the sum of a countable number of star-countable open coverings can be embedded into the product of a 0-dimensional metric space and the Hilbert fundamental cube. An open covering is called star-countable if every element of it intersects at most countable elements of it.

2) An open covering of a topological space is called star-finite if every element of it intersects at most finite elements of it. According to Morita [3] a topological space is called to have the star-finite property if every open covering of it can be refined by a star-finite open covering.

3) $\dim A$ denotes the covering dimension of A.

4) A topological space is called to have the Lindelöf property if every open covering of it has a countable subcovering. Morita [3] proved that every regular space with the Lindelöf property has the star-finite property.