44. On Metric General Connections

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In this note, the author will show that the Levi-Civita's connections of Riemann spaces can be generalized in the theory of general connections under some conditions on an n-dimensional differentiable manifold $\mathfrak{X}$. He will use the notations in [3].

1. A tensor P of type (1, 1) is called *normal* when P as a homomorphism of the tangent bundle $T(\mathfrak{X})$ of $\mathfrak{X}$ is an isomorphism on each $P(T_x(\mathfrak{X}))=P_x(\mathfrak{X})$, $x\in\mathfrak{X}$, and $\dim P_x(\mathfrak{X})$ is constant. Let us assume that P is normal and put $\dim P_x(\mathfrak{X})=m$. If we put $N_x(\mathfrak{X})=$the kernel of P on $T_x(\mathfrak{X})$, then we have

$$T_x(\mathfrak{X})=P_x(\mathfrak{X})+N_x(\mathfrak{X}).$$

According to the direct sum decomposition of $T(\mathfrak{X})$, we define two projections A and N which map $T_x(\mathfrak{X})$ onto $P_x(\mathfrak{X})$ and $N_x(\mathfrak{X})$ respectively at each point x of $\mathfrak{X}$. A and N may be considered as tensors of type (1, 1) of $\mathfrak{X}$. Clearly we have $A+N=I$, $A^2=A$, $N^2=N$, $AN=NA=0$, $AP=PA=P$ and $NP=PN=0$, where I denotes the fundamental unit tensor of type (1, 1).

Now, we say that a normal tensor P is *orthogonally related with* a non-singular symmetric tensor $G=g_{ij}du^i\otimes du^j$, if $P_x(\mathfrak{X})$ and $N_x(\mathfrak{X})$ are mutually orthogonal with respect to G, regarding G as a metric tensor.

A general connection Γ, which is locally written as

$$\Gamma=\partial u_i\otimes(P^i_j d^2u^j+\Gamma^i_{jh}\,du^j\otimes du^h),^{1)}\quad \partial u_i=\partial/\partial u^i,$$

is called *normal*, if the tensor $P=\lambda(\Gamma)$[2] $=\partial u_i\otimes P^i_j du^j$ is normal.

A normal general connection Γ is called *proper*,[3] if the tensor of type (1, 2) with local components $N^i_k\Gamma^k_{jh}$ vanishes, where N^i_j are the local components of the tensor N.

We say that a general connection Γ satisfies *the metric condition* for a symmetric covariant tensor $G=g_{ij}du^i\otimes du^j$, if

$$(1)\qquad DG=g_{ij,h}du^i\otimes du^j\otimes du^h=0,$$

where DG denotes the covariant differential of G with respect to Γ.[4] On the metric condition, the following theorem holds good as in the

1) See [3].

2) See [3], §2.

3) On the geometrical meaning of this condition, see Theorem 5.2 of [4]. In general, Γ^i_{jh} are not local components of a tensor of type (1, 3) as the classical affine connections but $N^i_k\Gamma^k_{jh}$ are so.

4) See (2.15) of [3].