68. On Köthe's Problem concerning Algebras for which Every Indecomposable Module Is Cyclic. I[1]

By Yutaka KAWADA
Mathematical Institute, Gunma University

(Comm. by K. SHODA, M.J.A., June 12, 1961)

§1. **Introduction.** In 1934, in connection with his theory, G. Köthe [2] proposed the problem to determine the general type of ring A (with a unit and satisfying the minimum condition) possessing the property that every finitely generated indecomposable left or right A-module is cyclic. A ring or algebra with this property will be called a Köthe ring or Köthe algebra. Köthe himself solved this problem for the special case of commutative rings. As for non-commutative rings, he proved only that uni-serial rings are Köthe rings.

In 1941, T. Nakayama [5, 6] introduced the notion of generalized uni-serial rings as a generalization of uni-serial rings, and proved that generalized uni-serial rings are Köthe rings. However, as is shown by Nakayama, the rings of this type are not general enough for solving Köthe's problem.

Recently H. Tachikawa [8] has called an algebra A an algebra of cyclic-cocyclic representation type, if any finitely generated indecomposable left (resp. right) A-module is either homomorphic to an indecomposable left (resp. right) ideal of A generated by a primitive idempotent, or isomorphic to a submodule of an indecomposable injective left (resp. right) module, and he has determined the structure of such algebras. However, algebras of cyclic-cocyclic representation type are not always Köthe algebras.

Thus the most general type of Köthe rings known hitherto is generalized uni-serial rings, and any class of rings which contains non-commutative rings and for which the solution of Köthe's problem is given seems to have never been obtained in the literature.[2]

The purpose of this paper is to announce that Köthe's problem mentioned above is completely solved for the case of self-basic algebras.[3] As is well known, every commutative algebra is self-basic.

---

1) The results of this paper were reported by the author at the meeting of Math. Soc. of Japan, held in October, 1960.

2) In case A is an algebra over an algebraically closed field and the square of its radical is zero, T. Yoshii [10] has given some sufficient conditions for A to be a Köthe algebra.

3) An algebra (resp. ring) A is called a self-basic algebra (resp. ring) if A is the basic ring of A itself.