112. *Strongly p-Parabolic Systems*

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1. *Introduction.* As we know, I. G. Petrowsky [1] defined the p-parabolic single equation ($p\geqq 2$, integer) as follows:

$$\left[\left(\frac{\partial}{\partial t}\right)^m+\sum_{((k))}a^{(k_0,k)}\left(\frac{\partial}{\partial t}\right)^{k_0}\left(\frac{\partial}{\partial x_1}\right)^{k_1}\cdots\left(\frac{\partial}{\partial x_n}\right)^{k_n}\right]u$$
$$+\sum_{(k)}b^{(k_0,k)}\left(\frac{\partial}{\partial t}\right)^{k_0}\left(\frac{\partial}{\partial x_1}\right)^{k_1}\cdots\left(\frac{\partial}{\partial x_n}\right)^{k_n}u=f(t,x_1,\cdots,x_n), \tag{1.1}$$

1) $k_0<m$, $k=(k_1,\cdots,k_n)$; $pk_0+k_1+\cdots+k_n\leqq pm$, and where $\sum_{((k))}$ and $\sum_{(k)}$ mean the summations of the terms; $pk_0+k_1+\cdots+k_n=pm$, and the terms; $pk_0+k_1+\cdots+k_n<pm$, respectively.

2) The roots $\lambda_i(\xi)$ of the characteristic equation

$$\lambda^m+\sum_{((k))}a^{(k_0,k)}\lambda^{k_0}(i\xi)^k=0$$

satisfy

(1.2) real part $\lambda_i(\xi)\leqq-\delta|\xi|^p$, for some positive constant δ,

where $\xi=(\xi_1,\xi_2,\cdots,\xi_n)$, $(i\xi)^k=(i\xi_1)^{k_1}\cdots(i\xi_n)^{k_n}$, and $|\xi|=\left(\sum_{i=1}^n\xi_i^2\right)^{1/2}$.[1]

Petrowsky [1] proved that this forward Cauchy problem for this equation is well posed. On the other hand, he showed that, if there exist $\xi^0\neq 0$ (real) and $\lambda_i(\xi)$ such that

(1.3) real part $\lambda_i(\xi^0)>0$,

then the forward Cauchy problem for (1.1) is never well posed.[2]

Is the condition (1.2) necessary? This is not true. However, we want to show that the condition (1.2) is necessary, when we restrict ourselves to the strongly well posed equations (whose definition is given by definition 1.1).

Definition 1.1. *We say that* (1.1) *is strongly well posed* (*as a p-evolution equation*), *if* (1.1) *is well posed for any choice of the lower part*: what we call the lower part is

$$l\left(\frac{\partial}{\partial t},\frac{\partial}{\partial x_1},\cdots,\frac{\partial}{\partial x_n}\right)u=\sum_{(k)}b^{(k_0,k)}\left(\frac{\partial}{\partial t}\right)^{k_0}\left(\frac{\partial}{\partial x_1}\right)^{k_1}\cdots\left(\frac{\partial}{\partial x_n}\right)^{k_n}u.$$

Our purpose is to prove the following.

Theorem 1.1. *In order that the p-evolution equation* (1.1) ($p\geqq 2$,

1) It is clear that such a positive integer p satisfying the condition (1.2) must be even.

2) He proved this fact in the case where the coefficients depend only on t for the p-evolution systems.