102. Remarks on Cantor's Absolute. II

By Gaisi TAKEUTI
Department of Mathematics, Tokyo University of Education, Tokyo
(Comm. by Z. SUETUNA, M.J.A., Oct. 12, 1961)

As to the notions and notations we refer to [1, 2] throughout this paper. In [2], the author presented Cantor's Absolute as a universe satisfying certain conditions. In addition to these conditions we shall now assume the following:

For any set a and any well-defined univalent relation, there exists a set consisting of all sets each of which corresponds to an element of a by the relation.

In assuming this, we shall prove in this paper that, for every definable class of true closed formulas in T_C, a formula with the following meaning belongs to T_C:

For every set a of C, there exists a set b which has a as an element and which is a super-complete model of all formulas of the class under consideration.

More exactly, this assertion is given in the following form, if we use $\mathfrak{A}$ and $\mathfrak{D}$ in the same meaning as in [2]:

($*$) $\forall u \exists x(u \in x \wedge \forall y \forall z(y \in x \wedge (z \subseteq y \vee z \in y) \vdash z \in x) \wedge \forall y(\mathfrak{A}(y) \vdash \mathfrak{D}(x, y)))$.

This is an extension of the problem (B) in [2], and we left (B) as an open problem.

First we shall define some concepts. Let us extend the notion of 'set theory' in [2] to a set theory in the first order predicate calculus, which consists of not only the predicate ε, logical symbols and bound variables, but finitely or infinitely many individual constants. If T is such a set theory which contains $a_0, a_1, \cdots$ as indivisual constants, we call T a set theory with $a_0, a_1, \cdots$.

Let T be a set theory with $a_0, a_1, \cdots$ and B_T be the class consisting of all $\{x\}\mathfrak{A}(x, a_0, a_1, \cdots)$, where $\{x\}\mathfrak{A}(x, a_0, a_1, \cdots)$ consists only of logical symbols, the predicate ε, bound variables and $a_0, a_1, \cdots$, and it will be abbreviated as $\{x\}\mathfrak{A}(x)$ if no confusion is to be feared. T is called to be 'definite', if it satisfies the following conditions:

1) T is complete.

2) If $\exists x \mathfrak{A}(x)$ belongs to T, then there exists a formula $\exists x \mathfrak{B}(x)$ such that $\exists x \mathfrak{B}(x)$, $\forall x \forall y(\mathfrak{B}(x) \wedge \mathfrak{B}(y) \vdash x=y)$ and $\exists x(\mathfrak{A}(x) \wedge \mathfrak{B}(x))$ belong to T. Let $\{x\}\mathfrak{A}(x)$ and $\{x\}\mathfrak{B}(x)$ belong to B_T. We define '$\{x\}\mathfrak{A}(x)$ belongs to the same class as $\{x\}\mathfrak{B}(x)$ with respect to T', if and only if $\forall x(\mathfrak{A}(x) \dashv\vdash \mathfrak{B}(x))$ belongs to T. The class which contains $\{x\}\mathfrak{A}(x)$ is written $(\{x\}\mathfrak{A}(x))$ and $\{x\}\mathfrak{A}(x)$ is said to represent the class. A class