1. The theorem. Let $X$ be a complete locally convex linear topological space, and $L(X, X)$ the algebra of all continuous linear operators on $X$ into $X$. A pseudo-resolvent $J_\lambda$ is a function on a subset $D(J)$ of the complex plane with values in $L(X, X)$ satisfying the resolvent equation

$$ (1) \quad J_\lambda - J_\mu = (\mu - \lambda)J_\lambda J_\mu. $$

We have, denoting by $I$ the identity operator,

$$ (2) \quad (I - \lambda J_\lambda) = (I - (\mu - \mu)J_\lambda)(I - \mu J_\mu) $$

and

$$ (3) \quad \lambda J_\lambda (I - \mu J_\mu) = (1 - \mu(\mu - \lambda)^{-1})\lambda J_\lambda - \lambda(\lambda - \mu)^{-1}\mu J_\mu. $$

We see, by (1), that all $J_\lambda, \lambda \in D(J)$, have a common null space $N(J)$ and a common range $R(J)$. We also see, by (2), that all $I - \lambda J_\lambda, \lambda \in D(J)$, have a common null space $N(I - J)$ and a common range $R(I - J)$. $N(J)$ and $N(I - J)$ are closed linear subspace of $X$, but $R(J)$ and $R(I - J)$ need not be closed; we shall denote by $R(J)^\sigma$ and $R(I - J)^\sigma$ their closures respectively.

To formulate our ergodic theorems we prepare two lemmas.

**Lemma 1.** Let there exist a sequence $\{\lambda_n\}$ of numbers $\in D(J)$ such that

$$ (4) \quad \lim_{n \to \infty} \lambda_n = 0 \text{ and the family of operators } \{\lambda_n J_{\lambda_n}\} \text{ is equi-continuous}. $$

Then we have

$$ (5) \quad R(I - J)^\sigma = P(J) = \{x \in X; \lim_{n \to \infty} \lambda_n J_{\lambda_n} x = 0\}, $$

and hence

$$ (6) \quad N(I - J) \cap R(I - J)^\sigma = \{0\}. $$

**Lemma 1'.** Let there exist a sequence $\{\lambda_n\}$ of numbers $\in D(J)$ such that

$$ (4)' \quad \lim_{n \to \infty} |\lambda_n| = \infty \text{ and the family of operators } \{\lambda_n J_{\lambda_n}\} \text{ is equi-continuous}. $$

Then we have

$$ (5)' \quad R(J)^\sigma = I(J) = \{x \in X; \lim_{n \to \infty} \lambda_n J_{\lambda_n} x = x\} $$

and hence

$$ (6)' \quad N(J) \cap R(J)^\sigma = \{0\}. $$

Our ergodic theorems read as follows.

**Theorem 1.** Let (4) be satisfied. Let, for a given $x \in X$, there exist a subsequence $\{\lambda_{n_k}\}$ of $\{\lambda_n\}$ such that