128. On Certain Reduction Theorems for Systems of Differential Equations which Contain a Turning Point

By Kenjiro OKUBO
Department of Mathematics, Tokyo Metropolitan University
(Comm. by K. KUNUGI, M.J.A., Nov. 13, 1961)

1. **Introductions.** In this paper we consider a system of linear ordinary differential equations

(1.1) $$\varepsilon dx/dt = A(t, \varepsilon)x,$$

where x is an n-vector: $A(t, \varepsilon)$ is a matrix of type (n, n), which admits a uniformly asymptotic expansion

(1.2) $$A(t, \varepsilon) = \sum_{j=0}^{\infty} A_j(t)\varepsilon^j$$

for $|t| < t_0$, as ε tends to zero through a domain $|\arg \varepsilon - \theta| < \varepsilon_0$. The coefficients of this expansion, $A_j(t)$ are holomorphic functions of t in the domain $|t| < t_0$.

The system has a turning point at the origin, if $A_0(t)$ has a set of eigenvalues: $\lambda_{j_1}(t), \cdots, \lambda_{j_p}(t) (p \leq n)$, which are zero for $t=0$, but at least a pair of eigenvalues are not identically equal, where, by a theorem due to Sibuya, (cf. Sibuya, Y. [3]), we may assume $p=n$.

Though a general method to treat such a system is not yet known, all the known results are obtained by reducing the coefficient matrix $A(t, \varepsilon)$ to a matrix, whose elements are polynomials in the independent variable. Moreover, if there is a formal transformation

(1.3) $$y = P(t, \varepsilon)x \quad P(t, \varepsilon) \sim \sum_{j=0}^{\infty} P_j(t)\varepsilon^j$$

such that

(1.4) $$\det P_0(0) \neq 0, \quad P_j(t)\text{: holomorphic for } |t| < t_0$$

which reduces the system (1.1) to a system with polynomial coefficients, then, in a sectorial domain, there is a matrix $Q(t, \varepsilon)$ which has the same asymptotic expansion as $P(t, \varepsilon)$. (cf. Sibuya, Y. [4]). We shall call a formal transformation (1.3) with the properties (1.4), a *formal admissible transformation.*

Our results are stated in two theorems:

Theorem 1. *If in* (1.2) $A_0(t)$ *is in the form*

(1.5.1) $$A_0(t) = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & & \cdots & & \\ 0 & 0 & 0 & \cdots & 1 \\ t & 0 & 0 & \cdots & 0 \end{pmatrix},$$

then there is a formal admissible series (1.3) *such that*

$$\varepsilon dy/dt = A_0(t)y.$$