127. Remarks on Metric Spaces with U-extension Properties

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In this paper, a space S is uniform and a function is real and, unless otherwise specified, uniformly continuous. A sequence $\{A_n\}$ of subsets is said to be *discretely normally separated* by a sequence $\{B_n\}$ of subsets if $\{B_n\}$ is discrete and, for every n, there is a function f, $0\leq f\leq 1$, on S with the value 1 on A_n and 0 on $S-B_n$; $\{A_n\}$ is *uniformly separated* by $\{B_n\}$ if there is an entourage U with $B_n\supset U(A_n)$ for every n; $\{A_n\}$ is *U-discrete*, U being an entourage, if, for any point $x\in S$, $U(x)$ meets at most one member of the sequence; finally, $\{A_n\}$ is *uniformly discrete* if it is U-discrete for some U.

0. Let us give our attention to the similarity between the following two conditions:

(UC) *Any sequence $\{A_n\}$ of subsets discretely normally separated by some sequence $\{B_n\}$ of subsets is uniformly separated by $\{B_n\}$*, which is a necessary and sufficient condition in order for any continuous real function on a space to be uniform [1, Theorem 1].

(E) *Let $\{A_n\}$ be a U-discrete sequence of subsets, and $\{a_n\}$ a sequence of natural numbers, then there is an entourage V with $V^{a_n}(A_n)\subset U(A_n)$ for every n*, which is a necessary and sufficient condition in order for any function defined on any uniform subspace of a space S to have a uniform extension to S [2]. When $\{A_n\}$ is a U-discrete sequence and $W^4\subset U$, then $\{A_n\}$ is discretely normally separated by the sequence $\{W(A_n)\}$.

In a metric space, we have several conditions equivalent to (UC) [3, Theorem 1], while we have a condition corresponding to (E) in which V^{a_n} in (E) is replaced by $V^\infty=\bigcup_m V^m$ [4, Lemma 2]. Therefore it is natural to seek the conditions of metric spaces equivalent to (E) corresponding to each of the known conditions equivalent to (UC).

1. From here on, we will consider a space metric and complete (a metric space satisfying (UC) is necessarily complete). $V_{1/n}$ is the entourage of the space consisting of pairs of points whose distances apart are less than $1/n$, a V_e-discrete family, e being a positive number, is simply said to be *e-discrete*, and a space satisfying (E) is said to have a *u-extension property*. We know [4, Theorem 2] that *a space S has a u-extension property if and only if for any natural number n there is a compact subset K such that, for any open subset G containing K, there is a natural number m satisfying*