126. *Operator-Valued Entropy of a Quantum Mechanical Measurement**)

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The subject to be discussed is, from the point of view of Schrödinger theory, finding suitable measures for loss of definition of a state. From the point of view of hilbert space operator theory, it is simply finding suitable measures for the "extent of non-commutativity" of an operator D with the algebra generated by a discrete family of projections $\{E_i\}$. From the latter point of view, the interest of the paper is concentrated in §2.

In addition to the conspicuous lack of generality of the hypotheses, the statements proved here are incomplete in other respects. Suggestions for future research are accordingly made at several points.

1. States (Segal's approach) and entropy. In the usual Schrödinger theory, one may consider every state of a system to be the set of expectation values it assigns to observables: any state is a certain functional. The (vector) pure state corresponding to the wave-vector $\psi\in\mathcal{H}$ is the functional mapping every bounded hermitian A—'observable'—to the number $(A\psi,\psi)$; the mixed state corresponding to probabilities λ_i respectively of wave-vectors ψ_i ($\lambda_i\geqq 0, \sum\lambda_i=1$), maps A to $\sum\lambda_i(A\psi_i,\psi_i)$. The phases of ψ and ψ_i being irrelevant here, one may prefer to associate to the pure state, not the vector ψ, but instead the operator P of projection on the subspace $[\psi]\subset\mathcal{H}$; to the mixed state then, will be associated the operator $D=\sum\lambda_iP_i$, where P_i is the projection on $[\psi_i]$. The functional then takes A to $\mathrm{tr}\,(PA)$, respectively $\mathrm{tr}\,(DA)$. It is a simple and well-known, but important, fact that there is no loss of generality in assuming here that the P_i are orthogonal—D is after all simply an arbitrary positive (semi-) definite operator with trace 1.

Now I propose to seek a definition of entropy-increase which will measure the extent to which a state is made "more mixed" by being subjected to a measurement [11]. If the dials of the measuring instrument are read, the "packet is reduced", and entropy should decrease. If the dials are not read, the measurement replaces any

*) Presented in preliminary form at Stevens Institute of Technology, 21 March 1961, and in toto at the University of Chicago, 16 August 1961. Theorems 2 and 3 were mentioned as new in the former talk because I had not learned of the then unpublished paper of Nakamura and Umegaki [8].