140. Some Characterizations of Fourier Transforms. II

By Koziro IWASAKI
Musashi Institute of Technology, Tokyo

(Comm. by Z. SUETUNA, M.J.A., Dec. 12, 1961)

1. In the theory of the Fourier exponential transform on the real number field $\boldsymbol{R}$ the following four properties play important roles. Namely,

a) the Fourier exponential transform

$$E:\varphi(x)\rightarrow E\varphi(x)=\int_{-\infty}^{\infty}e^{2\pi ixt}\,\varphi(t)\,dt$$

is a linear mapping from $\mathfrak{P}$ onto itself where $\mathfrak{P}$ is the space of all functions of class C^∞ whose derivatives are all rapidly decreasing,

b) $E(\varphi*\psi)=E\varphi\cdot E\psi$,

c) $\int_{\boldsymbol{R}}|E\varphi|^2\,dx=\int_{\boldsymbol{R}}|\varphi|^2\,dx$,

d) $\sum_{n\in \boldsymbol{Z}}E\varphi(n)=\sum_{n\in \boldsymbol{Z}}\varphi(n)$

where φ and ψ belong to $\mathfrak{P}$, $\varphi*\psi$ is the convultion of φ and ψ, and $\boldsymbol{Z}$ is the set of all integers.

Some years ago we have pointed out that the properties b) and d) characterize the Fourier exponential transform ([2]). In this paper we shall deal with another characterization. We denote $\varphi(x+a)$ with $\varphi_a(x)$ as a function of x.

Now the main result is as follows:

Theorem. *If there exists a linear mapping T from $\mathfrak{P}$ into the space of C^∞ functions on a Riemannian manifold $\mathfrak{R}$ satisfying the conditions:*

I) *when a function series $\varphi_1,\varphi_2,\cdots$ in $\mathfrak{P}$ converges to 0 by L^1-topology, the series $T\varphi_1, T\varphi_2,\cdots$ converges to 0 by L^∞-topology,*

II_1) *to any point ξ of $\mathfrak{R}$ and any open set U containing ξ there exists a function φ in $\mathfrak{P}$ such that the support of $T\varphi$ is contained in U and $T\varphi(\xi)$ is different from 0 and*

II_2) *to the same function φ $T\varphi_a(\xi)$* grad *$T\varphi(\xi)$ differs from $T\varphi(\xi)$* grad *$T\varphi_a(\xi)$ with some real number a* (here a may depend on φ),

III) $$T(\varphi*\psi)=T\varphi\cdot T\psi,$$

IV) $$\int_{\boldsymbol{R}}|T\varphi|^2\,d\xi=\int_{\boldsymbol{R}}|\varphi|^2\,dx,$$

then there is a C^∞ bijection r from $\mathfrak{R}$ to $\boldsymbol{R}$ such that

$$T\varphi(\xi)=E\varphi(r\xi).$$

Moreover if we assume an additional hypothesis