19. *On Infinitesimal Operators of Irreducible Representations of the Lorentz Group of n-th Order*

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§1. **Introduction.** The Lorentz group of n-th order is the connected component of the identity element of the group of such homogeneous linear transformations in the real n-dimensional vector space that leave the quadratic form $x_1^2+x_2^2+\cdots+x_{n-1}^2-x_n^2$ invariant. We shall denote it by L_n.

In the present paper, we investigate differentiable irreducible representations by bounded (not necessarily unitary) operators in a Hilbert space. We shall make use of what is called the infinitesimal method.

First we establish the system of commutation relations which must be fulfilled by the corresponding infinitesimal operators. Next we give a class of the solutions of these operator equations. It is believed that there exist no other solutions, but the proof of this fact is not completed. In another paper [6] we shall classify irreducible representations and distinguish unitary ones.

The same problem has been discussed for the case $n=5$ by L. H. Thomas [1] and T. D. Newton [2] and for the case $n=4$ by M. A. Naimark [3]. The results in the present paper and in [6] may throw light on the problem of explicite construction of irreducible unitary representations of these groups and suggest the existence of integrable irreducible unitary representations when n is odd.

§2. **Lie algebra of L_n.** Consider in L_n $n(n-1)/2$ one-parameter subgroups of the following types:

$$g_{ij}(t)=\begin{array}{c}\qquad (i)\qquad\quad (j)\qquad\qquad \\ \left\|\begin{matrix} 1 & & & \\ & \ddots & & \\ & & \begin{matrix}\cos t\cdots\sin t\\ \cdots\cdots\cdots\\ -\sin t\cdots\cos t\end{matrix} & \\ & & & \ddots \\ & & & & 1\end{matrix}\right\|\end{array},\quad g_k(t)=\begin{array}{c}\qquad\qquad (k)\qquad\qquad \\ \left\|\begin{matrix} 1 & & \\ & \ddots & \\ & & 1 \\ & & & \begin{matrix}\cosh t\cdots\sinh t\\ 0\quad\cdots\quad 0\\ \cdots\cdots\cdots\\ \sinh t\cdots\cosh t\end{matrix}\end{matrix}\right\|\end{array}, \tag{1}$$

where $1\leq i,j\leq n-1, 1\leq k\leq n-1$.

The matrix $g_{ij}(t)$ corresponds to a rotation in the plane (x_ix_j) and the matrix $g_k(t)$ corresponds to a hyperbolic rotation in the plane (x_kx_n).